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PARTIAL CONTENTS

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CONTENTS

IGR TRANSLITERATION OF RUSSIAN

The AGI Translation Office has adopted the essential features of Cyrillic transliteration recommended by the U. S Department of the Interior Board of Geographic Names, Washington D. C.

Alphabet		transliteration
А	а	a
Б	б	b
В	в	v
Г	г	g
Д	д	d
Е	е	e, ye (1)
Ё	ё	ё, yё
Ж	ж	zh
З	з	z
И	и	i (2)
Й	й	y
К	к	k
Л	л	l
М	м	m
Н	н	n
О	о	o
П	п	p
Р	р	r
С	с	s
Т	т	t
У	у	u
Ф	ф	f
Х	х	kh
Ц	ц	ts
Ч	ч	ch
Ш	ш	sh
Щ	щ	shch
Ъ	ъ	" (3)
Ы	ы	y
Ь	ь	' (3)
Э	э	e
Ю	ю	yu
Я	я	ya

However, the AGI Translation Office recommends the following modifications:

1. Ye initially, after vowels, and after ъ, ь. Customary usage calls for "ie" in many names, e. g., SOVIET KIEV, DNIEPER, etc.; or "ye", e. g., BYELORUSSIA, where "e" follows consonants. "e" with dieresis in Russian should be given as "yo".
2. Omitted if preceding a "y", for example, Arkhangelsky (not "iy"; not "ii").
3. Generally omitted.

NOTE: Well-known place and personal names that have wide acceptance will be used. Some translations may include elements of previous German transliteration from the Russian; this occurs in IGR most commonly in maps and lists of references. The reader's attention is called to the following variations between German and English systems which may cause confusion when trying to check back to original Russian sources.

German	English
w	v
s	z
ch	kh
tz	ts
tsch	ch
sch	sh
schtsch	shch
ja	ya
ju	yu

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THE CONCEPT OF FACIES (PART 1 OF 3)[1]

by V. P. Markevich

• *translated by Ivan Mittin*[2] •

EDITOR'S NOTE

This work will be published in three parts, tentatively to appear in three successive issues of International Geology Review. The work contains an Introduction, five chapters, and a list of references to appear as follows:

INTRODUCTION

It is well known that the sedimentary complex of rocks which forms the upper portion of the earth's crust is not uniform in composition and structural characteristics. A considerable portion of the earth's surface is covered with sedimentary rocks of varied composition, deposited in layers of different thicknesses.

It has been noted for a long time that layers of sedimentary rocks differ from each other, in some instances very sharply, by their composition, structural-textural features and content of organic fossils. It has also been noted that each layer of sedimentary deposits taken separately exhibits such modifications areally; in each case these modifications are expressed differently; they take place abruptly or gradually. A great Russian scientist, M. V. Lomonosov, wrote about such modifications as early as the second part of the XVIIIth century. In his famous work "Concerning the earth's strata" (1757-1759), M. V. Lomonosov, in describing the sediments of one coal deposit in England, noted that "the layers are often interbedded with gray stones of various colors, with clay and other minerals in layers and mixtures. The number of layers is indefinite and not [all are] of [the] same order. Sometimes shale and coal are separated by a layer of a definite rock or sandstone, sometimes they are in contact with each other" (publ. 1949, p. 39). Further in the same work he writes: "Mountain coal and shale lying in adjoining layers are sometimes so intermixed that it is difficult to separate them. Furthermore, shales differ extremely in color, by the degree of hardness [and] by their composition."

Thus, M. V. Lomonosov paid considerable attention to the study of modifications which sedimentary layers undergo in space and time. Subsequently these changes became known as "facies."

The term "facies" from (Latin facies---a face, image, type) was first introduced in the literature by N. Steno in 1669. He used this term for the six thick series of sedimentary beds that he differentiated in a section of deposits in the vicinity of Tuskany and applied this term [in a] geochronological [sense].

In 1838 a work by A. Gressly, who had studied changes of lithology in the Jurassic beds of eastern France, was published. In his work, A. Gressly tried to limit the definition of the term facies by providing it with a definite meaning. He considered changes in the layers of sedimentary deposits to be a great importance, as had M. V. Lomonosov at an earlier date. A. Gressly concluded that "each deposit within the area of its areal distribution [exhibits]

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[1] Translated from Ponyatie "Fatsiya." Akademii Nauk SSSR, Moskva, 1957. Edited by E. D. McKee and Curt Teichert, U. S. Geological Survey. Published with the permission of the Director, U. S. Geological Survey. Parts 2 and 3 of this work will be published in succeeding issues of International Geology Review.

[2] U. S. Geological Survey.

quite definite variations; these variations are constant specific features both in their petrographic composition and also in the paleontologic features of their fossils, their changes being governed by definite and fixed laws" (Nalivkin, 1932, p. 5). In order to designate changes of this type in contemporaneous deposits, A. Gressly applied the term "facies."

A Gressly has defined a facies as "the collective character of various deposits expressed in [terms of] petrographic, geognostic, or in fact, paleontologic features" (Litologicheskii Sbornik, 1, 1948, p. 16).

It should be noted, however, that A. Gressly himself did not adhere to his definition of the term facies, and used this term in an entirely different sense, by segregating, for instance, fresh-water, brackish-water, marine and similar facies and subfacies. Essentially, A. Gressly considered all rock features as facies features, both petrographic and geologic, and particularly paleontologic to which he ascribed a decisive value.

For a long time following A. Gressly, researchers did not attempt to make the term "facies" more accurate, and employed it in the manner suitable to themselves giving the term various meanings.

The term "facies" was introduced in the Russian literature in 1869 by N. A. Golovkinskii who, in our opinion, understood correctly the value of this term in geology and, in contrast with his predecessors, put emphasis on the change of facies not only along a stratum but through its vertical section. N. A. Golovkinskii was first to show that different horizons may intersect at acute angles as the result of a change in facies composition of rocks, that is, the migration of facies in time and space.

Subsequently, different researchers defined the concept of facies quite dissimilarly and gave it various meanings. Thus, some researchers defined facies as sedimentary environments, others as environments or landscape units, still others as layers or rock suites, and a fourth group, as complexes of rock features.

A. A. Inostrantzev compared the term "facies" with a number of other terms of various meanings such as: "type", "area", "province." Thus, he writes: "The names facies, types, areas and provinces are all interpreted as horizontally distributed contemporaneous formations, with either paleontologic or petrographic differences." (Inostrantzev, 1872, p. 484).

On the other hand, A. A. Inostrantzev points out that " a given group of geologic formations can include freshwater, off-shore, marine, oceanic, etc., facies" (Ibid).

A similar interpretation of this term was given ten years later by J. Walther who said in his work that "different aspects of contemporaneously formed rocks are called facies." However, J. Walther also used the term facies in a different sense, regarding a facies as a lithologic expression of a sedimentary environment, or as physical conditions of the sea bottom. He understood the term facies in the broad sense and extended it to both sedimentary and igneous rocks, using as a basis a variety of rock features. A somewhat different interpretation of this term was given by E. Haug who proposes to retain its "original meaning" by A. Gressly, but yet immediately applies another definition. "We understand the term "facies," writes E. Haug, "as a combination of lithologic and paleontologic features of a layer at a designated place" (1938, p. 125). The essential difference in the definitions by A. Gressly and by E. Haug is obvious. A. Gressly understood the term facies as changes in the petrographic and paleontologic features observed within a layer, while E. Haug saw it as a combination of these features at a definite place within the layer.

CHAPTER I. THE CONCEPT OF "FACIES" IN THE SOVIET GEOLOGIC LITERATURE

A. A. Borisiak in his "Course of Historical Geology" (1922) pointed out that historical geology recognized "ancient facies" in every part of a rock and that "from the viewpoint of historical geology any layer of the earth's crust represents a definite facies." On the other hand the same author writes, "Usually the term facies designates the physical characteristics of a considered area or a section of the earth's surface, no matter whether land or sea bottom, which specifies a definite distribution of animals and plants; therefore, facies are characterized by the physical conditions, the fauna and the flora. Facies are characterized by lithologic properties of the layer in question and by its paleontologic remains" (1935, p. 18). Such a definition of the term facies differs considerably from the one by A. Gressly. In the first place, A. A. Borisiak identifies layers as facies, while A. Gressly used the term facies to designate the changes within the layer in the direction of its strike. A. A. Borisiak recognized the "physical characteristics of a given area or a section of the earth's surface" as recent facies, and "the lithologic properties of a considered stratum and its paleontologic remnants" as ancient facies.

Later D. V. Nalivkin devoted to the study of facies a special work which was published in 1932. He gave different definitions of recent and ancient facies. Thus, in his presentation, the recent facies is a part of the earth's surface which is characterized in its entire extent by uniform physico-geographical conditions and uniform fauna and flora; and "the ancient facies

is a part of a stratum, an entire stratum, or a series of strata, which in its entire extent has uniform lithologic composition and identical fauna and flora" (1932, p. 6).

The first of these definitions referring to the recent facies does not link the term with sediments. By defining the ancient facies as a part of a stratum, an entire stratum or a suite of strata, D. V. Nalivkin writes at the same time that "facies is a unit of the landscape." Naturally, such an indefiniteness in understanding the term facies brought about a reaction from many scientists. M. P. Kazakov, G. F. Mirchink, N. M. Strakhov, E. V. Shantzer in their joint work point out that the definition of facies by D. V. Nalivkin "does not convey the sense or meaning which this term has been given by historical geology," and moreover "in its most essential point." According to their opinion, "in historical geology facies is not concerned with rocks or fauna, but is a document of that physico-geographic environment in which the fauna and rocks were deposited. Historical geology aspires first of all to learn the precise nature of this environment concealed within the rocks." These researchers believe that "in historical geology the concept of facies is of a paleogeographical and not of a petrographical or faunal nature" (Kazakov and others, 1934, p. 455).

In our understanding, the drawback of D. V. Nalivkin's definitions of the term facies is the indefinite meaning given to this concept. As for rebuking D. V. Nalivkin for his not having given paleogeographical meaning to the term facies, this was not entirely just, for D. V. Nalivkin and correctly noted that the "study of facies is a natural introduction to paleogeography" and that "in considerable part it is a study of the conditions of sedimentation" (Nalivkin, 1932, p. 5). But he did not define the concept of facies precisely, nor did he give clear form to this definition.

In spite of the attempts of many a researcher to make the concept of facies more precise, this question has not yet been satisfactorily solved up to the present time.

So, in his work "Historical Geology" published in 1933, A. N. Mazarovich also applied the term facies in an indefinite manner, as reflected in the definitions which he gave to this term. He wrote: "Facies is a sum of petrographic and organic characteristics which define the individuality of the sections of the earth's surface illustrating their peculiar characteristics of deposition and population." But here again A. N. Mazarovich offered a different definition: "Otherwise, we call facies a definite area in which deposition of particular types of rock takes place and which is populated with organisms which depend entirely on local conditions" (Mazarovich, 1938, p. 47). Thus, A. N. Mazarovich employed the term facies very freely and gave it various meanings; according to his idea "facies could be considered both very broadly and very narrowly."

A. V. Kazakov in his work on the phosphate facies, after a short and hardly complete review of the definitions of the term facies by other researchers, wrote: "The geological facies first of all is a complex type of physico-geographic and oceanographic condition of accumulation and formation of deposits against a background of a definite biocoenosis. Classification of facies is not a rock classification. The paleogeographical and geochemical characteristics must become basic elements of the facies concept" (Kazakov, 1939, p. 33).

The above quoted definition of the concept of facies is obscure. The accumulation and formation of sediments does not depend entirely upon the conditions limited to the area of sediment accumulation in question. These processes are affected also by conditions which characterize adjacent sections of the earth's crust at different distances from the site of accumulation and formation of the sediments. It is not clear either what the author's concept is of the "paleogeographical characteristics." In his work, A. V. Kazakov employs the term facies rather freely, not confining himself to his definition of this concept, and he usually associates the term facies and the terms "deposition" and "rock." Thus, he writes: "Turning to the phosphorite deposits, we first of all become aware of the type and character of their bedding, genesis, the forming of phosphorite layers, their mineral paragenesis and also the paragenesis with adjoining facies.

"This multiform genetic complex of conditions clearly expressed along with the lithologic feature of the phosphorite rocks compels us to separate them into independent phosphorite facies of great paleogeographical significance. They serve as a document of the unique physico-geographical environment, one of the pages of the life of corresponding marine basins" (p. 33).

The quoted part does not clarify greatly the meaning of the "phosphorite facies" nor does it explain whether it is a "complex of conditions," or "documents of the physico-geographical environment," referring to rocks, or to sediments. In all probability, the author still considers facies as rocks enriched with phosphorites.

The introduction of the geochemical characteristic as a basic element of the facies concept is, as we see it, a step forward in making this concept more precise.

A. G. Eberzin (1940) introduced to the literature for the first time the term "lithofacies." Having generalized the data of his

investigations of the Pliocene deposits of the Black Sea region, he compiled a distribution map of the deposits of the Cimmerian strata on the basis of lithologic features and called it "Map of lithologic facies of the Cimmerian Basin" (fig. 1).

The above definitions have much in common with those given by D. V. Nalivkin and other investigators of the geological facies.

The definition of recent geochemical facies formulated by L. V. Pustovalov does not fully reflect either the peculiarities of the sedimentation process or the totality of physico-geographical and geochemical conditions which characterize the sedimentation process; furthermore, the biological factor as a reliable index of physico-geographical and physico-chemical conditions of the sediment-forming medium is not taken into account. It would be more appropriate to talk not about the geochemical facies but the geochemical characteristics, or geochemical features of the geological facies.

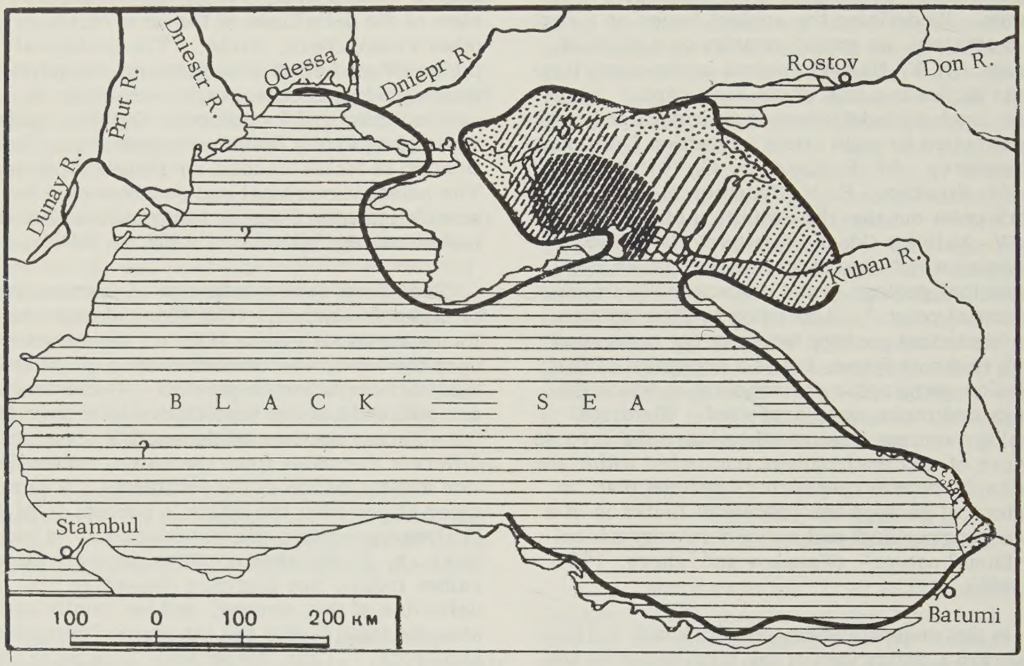

FIGURE 1. Map of lithologic facies of the Cimmerian basin (data from A.G. Eberzin, 1940).

clay; sandy clay; sand; clay with interbeds of ferruginous sandstones; sandstone, partially ferruginous; ore conglomerate.

L. V. Pustovalov gave the literature the terms: "Geochemical facies" and "terrigenous mineralogical facies." Analogous to the definitions of geological facies by D. V. Nalivkin, L. V. Pustovalov also gives to definitions of geochemical facies: one characterizes the recent geochemical facies, the other is for the ancient geochemical facies, with both definitions given more than one meaning.

L. V. Pustovalov considers the term geochemical facies as "a part of the earth's surface which for its entire extent is characterized by uniform physico-chemical and geochemical conditions of accumulation and formation of sedimentary rocks." The ancient geochemical facies is described by him as " a stratum or a suite of strata which in their entire extent possess the uniform original geochemical characteristics developed during the formation of sedimentary rocks; these characteristics become apparent first of all in the general occurrence of the same complex of syngenetic deposits which among themselves form related associations regulated by the physico-chemical conditions of the rock formation" (Pustovalov, 1940, pt. 1, p. 462).

As far as the definition of ancient facies is concerned, here first of all it is not clear why the author adds the "stratum or suite of strata" to geochemical facies. And what if the stratum itself is not characterized by "uniform primary geochemical features?" Moreoever, the very definitions of geochemical facies do not correspond to those geochemical facies which were distinguished by L. V. Pustovalov.

The facies differentiated by L. V. Pustovalov partially reflect the physico-chemical conditions of the medium in which the accumulation and formation of sedimentary material and the distribution of these or other syngenetic minerals

in the deposited material take place. In other words, L. V. Pustovalov's facies refer to conditions favorable for some chemical processes which may occur at various sections of the earth's surface and which regulate the conditions of appearance in a sediment of syngenetic minerals.

Thus, for instance, the hydrosulfide facies, the siderite facies and others do not indicate uniform physico-chemical and geochemical conditions of accumulation in sediments and formation of sedimentary rocks of the designated part of the earth's surface, but represent some peculiarities of the sedimentation medium in the vicinity of the deposits being formed.

Depending on the position (in relation to the sediment surface) of the oxidation-reduction limit, L. V. Pustovalov distinguished seven marine geochemical facies and arranged them in the order of the increase in oxidation potential. At the same time, the first, or hydrosulfide, facies is characterized by a lack of free oxygen, while others, such as oxidation and ultra-oxidation facies, on the contrary, are characterized by an excess of active oxygen. The remaining facies (siderite, chamoisite, glauconite and phosphorite) are characterized by intermediate indices of oxidation potential.

As for two other marine geochemical facies segregated by L. V. Pustovalov, the dolomite facies and the marine saline facies can not be included in the series of the first seven because different principles are applied to their segregation; they are separated on the basis of the degree and character of the salinity of the medium of sedimentation.

Finally, the continental facies have been segregated by L. V. Pustovalov on the basis of the weathering peculiarities of some rock types, as well as the conditions of the formation of mineral deposits, such as iron ores, coal, salt, and others. Unfortunately, the author has not given examples of geochemical paleofacies which according to him are rocks, that is, "a stratum or a suite of strata..."

Thus, there exists a discrepancy between the geochemical facies and L. V. Pustovalov's definition of this concept which leads to ambiguities in the understanding of geochemical facies.

The definitions of the geochemical facies given by L. V. Pustovalov refer on the one hand to sedimentary rocks in general, and on the other hand to sections of the earth's surface which are characterized by uniform physico-chemical conditions of accumulation and formation of the rocks. However, in separating definite geochemical facies, the facies is perceived as a zone in the environment corresponding to a given section of the earth's surface and affecting sedimentary processes.

In studying facies an important place must be given to the facies themselves and an equal if not greater place to the changes which characterize the transition from one type of facies into another and to the determination of a systematic pattern of these changes, whether the facies are geological or geochemical, lithological, etc. (according to various groups of investigators).

As for the term "terrigenous-mineralogical facies" introduced in the literature by L. V. Pustovalov (1947), it is not a large-scale concept. Introducing this concept, he wanted to indicate the influence of a depositional medium upon a homogeneous clastic source material. Thus, he writes: "Introducing the new concept of the terrigenous-mineralogical facies, we want by this term to emphasize that differences in the complex of the terrigenous components may be tied not only to various sources of the mineral supply...., but also to the fundamental changes in the composition of terrigenous components during the sedimentation process" (1947, p. 79).

The difference between the concepts of "terrigenous-mineralogical facies" and "terrigenous-mineralogical provinces" remains vague. The latter concept has been used in literature on a large scale. According to L. V. Pustovalov, the concept of terrigenous-mineralogical provinces is of a "purely static character and reflects only the presence in a pertinent complex of sediments of a definite combination of fragmental minerals," whereas the concept of the terrigenous-mineralogical facies "reflects the dynamics of the sedimentary process of accumulation and the changes in the character of the fragmental or clastic minerals which take place in the course of time" (1947, p. 80).

The quoted material permits one to think that the object in both cases is the same, but the discussion concerns different approaches to it. In the first case the actual distribution of the sedimentary material in the rock mass is objectively recorded, whereas in the second case the question concerns the dynamics of the process itself and its results. In our opinion, both of these aspects were sufficiently taken into account by V. P. Baturin (1937) and by other researchers in separating the terrigenous-mineralogical provinces, and therefore, in all probability there is no real need for introducing the new term in the literature.

V. V. Belousov also treats the term facies ambiguously. In one case he recognizes facies as "the composition of the deposits" (1948, p. 138), and on the following page of the same work he refers to facies and deposits as different categories and formulates that "the true distribution of the facies and the extensive area of homogeneous deposits is the result of some

gradual process..." In the same work one also finds a different interpretation of this term. Thus V. V. Belousov writes: "The conditions determining the extent of the final distribution of terrigenous material at the bottom of a basin are of extreme importance in understanding the processes of forming facies and facies zones" (p. 141). In this case the meaning of the term "facies" and still more the term "facies zone" is not clear.

M. S. Shvetzov in his published manual on the petrography of sedimentary rocks uses the term facies extremely seldom. But even in these rare instances, he gives a unique meaning to the term facies and does not attribute to it great significance. Thus, M. S. Shvetzov writes: "In the South Antarctic Ocean there has developed a glacial facies of dark silt low in $CaCO_3$, containing pebbles and striated boulders carried from the South Antarctic continent." In this case the participation of ice in transporting the sedimentary material is emphasized. Somewhere further on the same page, M. S. Shvetzov says: "The volcanic or tuffaceous silt which develops in the volcanic area can be separated from deposits of the typical blue silt, as well as from shallow water deposits, as a special facies." In this case the facies is understood to represent deposits.

As far as it is possible to judge from his work, N. M. Strakhov understands facies as associations of deposits. For instance, he writes: "Interpreting conditions of deposition of a facies of little thickness covering the center of a depression, we must proceed... from the assumption that the cross section of a facies of little thicknesses is stratigraphically equivalent to others in more westerly zones, for it contains the same horizons as they do though they are thinner..." and further [he says]: "Under such conditions, we are compelled to admit that the facies sediments of sections that are thin, which replace the reef zone transverse to the strike direction of stratification, were deposited under conditions of greater water depth than is represented by sediments of the reef zone" (1951, p. 311-312).

In this case, N. M. Strakhov applies the term facies in a broad sense, unifying in it the complex composition of those rocks that are characterized by relatively thin layers in contradistinction to other formations of the same age.

In the same work by N. M. Strakhov we read: "The study of a few detailed petrographic data shows that to all appearances, there are not any sharp petrographic or facies distinctions between the geosynclinal carbonate and the platform rocks" (p. 313).

Here, the petrographic and facies differences are treated by N. M. Strakhov as different categories.

Finally, from the N. M. Strakhov definition of formation,[3] as well as from his other statements, it follows that Strakhov attaches a broader meaning to the term facies than to formation or historical-geological environment. Thus, he writes: "The carbonate rocks of past geological ages commonly exist as great compact massifs which by all indications were formed under similar uniform facies conditions. Henceforth, we shall designate the large rock accumulations of this type as formations, but the conditions of their formation as historical-geological environments" (p. 238).

N. M. Strakhov has supplemented his work with several maps of the facies of coal deposits (compiled from V. N. Krestovnikov). These maps show areas of distribution of red, sandy-argillaceous, continental deposits, carbonate sediments, clays, sandstones, and others, as well as folded structures and geosynclinal areas. Thus, N. M. Strakhov applies the term facies to designate rock types that differ from each other by the conditions of their formation.

V. E. Khain in a series of his works applies the term facies to designate various rock types and their combinations, mostly by their lithologic characteristics. Separate varieties of rocks he calls lithofacies and their complexes he calls facies. The maps compiled from the description of these rocks are called, respectively, lithofacies and facies maps.

For instance, sandstones, conglomerates, breccias, limestones, and other rock types are separated as lithofacies, and the complexes of various rocks as facies. V. E. Khain and A. N. Shardonov use the term facies also in place of the term "deposits" [and] "rocks." Thus, they write: "In comparing facies of the Abon stage of the Eastern Caucasian region with the Overz facies, we note that as a whole they continue to retain a terrigenous character" (Khain and Shardonov, 1952, p. 53).

V. E. Khain in his work "Geotectonic bases for oil prospecting" published in 1954, has given considerable space to the analysis of the term facies and to allied terms. Here he correctly notes that the term facies has been subject to "contradictory interpretation and application (p. 89). However, the many explanations advanced by its author regarding the term facies are neither convincing nor clear.

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[3] It should be noted that the Russian word "formatsiya" stands for a concept quite different from the stratigraphic formation of American usage. See last sentence of this paragraph - E.D. McK., C.T.

In our opinion, the definition of the term facies presented in V. E. Khain's work cannot be accepted and is hardly in conformity with his further discussion of the matter. According to V. E. Khain, "facies is a reflection of the physico-geographical (hydro- or aerodynamic, geochemical climatic, and others) and geotectonic conditions of sedimentation during the early history of a rock's components and other peculiarities of the rock originated from this sediment" (p. 101).

According to his definition, facies is not a sediment, nor a condition of sedimentation, but something of abstract nature--" the reflection of conditions." Such a definition of facies suggests another vague definition of the term lithofacies by the American researcher, R. C. Moore (1953, p. 57, 60).

In our opinion, facies is not the reflection of a sedimentary environment. It would be more proper to say that in studying facies, we perceive the conditions of the sediment's formation. The second part of V. E. Khain's definition of early history of the sediment's components and of other rock properties is not clear either, though in studying facies these questions may be solved to some extent.

One should agree with V. E. Khain that "the concrete practical application of the definition of facies is not always easy" (p. 101). Contrary to his own definition and agreeing with N. B. Vassoevich and others, V. E. Khain writes: "in the first place it is necessary to fix clearly the boundaries between the two aspects of the term "facies", -- facies as a combination of the peculiarities of the sediments, and the facies as a medium of sediment accumulation ... thus, the latter may be interpreted in two ways: as the place of sediment deposition (in a limited sense) and as a combination of an erosional, a transportational and a depositional area (in a broad sense). In accord with Y. A. Zhemchuzhnikov, the term facies can be associated with conditions of sedimentation (river channels, sea shore lines, lakes, marshes, and others), and the sedimentation medium should be called the facies environment" (p. 101). This quotation reflects the desire of its author to combine various contradictory considerations of different researchers.

The advantage of V. E. Khain's attempt to use the term tectofacies offered by American researchers (L. Sloss, W. Krumbein, E. Dapples, see next chapter) is wholly obscure. Still more obscure is the meaning of the new terms used by him in his work, such as "anticlinal, synclinal, prefaulting, and other tectofacies" (p. 102).

The author's definitions of lithofacies also do not conform to the definition of facies. Thus, V. E. Khain writes: "Turning now from facies and facies environments to the rocks themselves and to the primary and inherited characteristics of these rocks, we shall note that the combination of these characteristics can best be unified in the concept of lithofacies" (p. 103). It follows from this passage that the lithofacies is understood to be "the combination of the rock characteristics." The author continues further: "in separating different lithofacies, the dominant, determining characteristics are lithological..." (p. 103). The latter passage is contradictory to the former because here the lithofacies are not used in the sense of "the combination of the rock characteristics" but as the rocks themselves.

It seems to us, that the quoted passages from the work of V. E. Khain sufficiently show that his definitions of the term facies and other related terms do not conform with one another, and do not correspond to those facies which he separates in practice.

In 1945-46 in the VNIIGRI in Leningrad several reports on the study of facies were presented. According to published data, the speakers were far from being in accord, and their materials and arguments did not introduce significant clarity to the concept of facies and, in some instances, they even made this question more complicated.

Some interesting material on the history and evolution of the term facies was cited in a report by N. B. Vassoevich. He properly notes in an analysis of this question that the term facies in its introduction to the literature and especially later has been used very loosely and has not been confined to a definite subject. N. B. Vassoevich proposed to introduce several new terms. He suggested a new term "signatziia" for free use to designate "a combination of rock characteristics, with facies significance." N. B. Vassoevich describes facies as the environment and condition of rock formation; in his mind "facies is the cause and signatziia is the effect" (1948, p. 36).

N. B. Vassoevich subdivides facies into highly arbitrary stages of rock development, or, as he says "the facies of environment of rock for the formation." Thus, he distinguishes:

1. "Origofacies" - "the facies of the primary depositional environment."

2. "Lapidofacies" - "the facies of the diagenetic environment."

3. "Densofacies" - "the facies of the metamorphic environment."

4. "Exedofacies" - "the facies of the weathering environment."

Then, N. B. Vassoevich proposes to introduce

the term "amphios" as a synonym for the term facies. He fails to define this term and points out that: "As a basic taxonomic unit in the study of the origofacies, it is possible to suggest a special term amphios (from Greek: Amphis - the medium, surrounding), as a synonym of facies in the sense characteristic of the work by A. Gressly (but not as interpreted by the Swiss geologist)" (1948, p. 35). This definition hardly explains the meaning of the term "amphios."

The meaning and contents of the term "origosignatziya" offered by N. B. Vassoevich is still less clear. "Origosignatziya", as he writes, refers to those primary properties of the sediments inherited by the rock through which the amphios (origofacies) can be recon - structed " (p. 36). According to N. B. Vasso-evich, the origosignatziya is an ancient origo-facies, the latter having been developed under environment of the land and sea. In his opinion, the signatziya is represented in the rocks, or rock suites. He renamed the ancient facies of D. V. Nalivkin as origosignatziya and the modern (recent) facies as origofacies. Thus, N. B. Vassoevich made an attempt to eliminate confusion in the understanding of the term facies by introducing new terms for designating the conditions and various processes associated with sediment accumulation, and the formation and history of sedimentary rocks. Such a solution of the problem can hardly be success-ful; on the contrary, it makes the problem more complicated. In his other later work (1951), N. B. Vassoevich did not utilize those terms and suggested a number of new ones, such as facies "of the first order," "of the second order," "of the third order," of the fourth order," the "katenada," and "vydel." In this work he defines facies as "the combina-tion of various interlinking, physico-geographic conditions which characterize a given section of the earth's surface (or did characterize it in the geologic past)" (1951, p. 13).

Considering the broad concept of facies, N. B. Vassoevich proposes also to differentiate more narrow concepts. He suggests the follow-ing terms: "Facies of the first order" is the areally smallest section which exhibits charac-ters of a specific combination of physico-geo-graphic, geochemical, bionomical, and other conditions setting up a qualitatively uniform environment of positive, zero, or negative sedimentation." This definition may be con-sidered a more developed variation of the above general definition of the concept of facies. It is not clear how the author can delineate areas of zero and negative sedimentation and isolate small sections on these areas calling them "facies of the first order." In a second category N. B. Vassoevich places "facies of the second order," i. e., "the combination of a number of different but adjacent and co-existing facies of the first order common with respect to certain essential condtions of the medium" (p. 13).

In the opinion of N. B. Vassoevich this principle allows isolation of facies into the following orders: third, fourth, and others.

Besides the terms mentioned, N. B. Vassoe-vich suggests some others. Thus, he proposes that "the combination of all synchronous facies in a definite area of sedimentation" should be designated by the term "Katenada". In his opinion "Katenada" may be "the facies of second order, or more often, the facies of a higher order such as the third or the fourth," in other words by Katenada or facies of the third order he refers to different areas within a marine basin, such as littoral, bathyal, and abyssal. Is it necessary? In our opinion, the extension of the term facies to this category of concepts is hardly expedient.

Finally, N. B. Vassoevich proposed another term "Vydel" as well as a number of technical terms related to the meaning that he puts into this concept.

The "Vydel", according to N. B. Vassoevich, is a sediment (or rock) or a series of adjacent sediments (or rocks) "considered as the pro-ducts of sedimentary processes, produced in one or several facies." N. B. Vassoevich proposes to distinguish several categories of "Vydels": elementary, summary-elementary (double, triple), compounds of the first order, and others.

It is very obvious that the terms introduced by N. B. Vassoevich in his earlier work (Litolo-gicheskii sbornik, 1, 1948) have not been utilized by him subsequently. As for the new terms, they have not, in our opinion, been explained with sufficient clarity either.

It is to be noted that the question of terminol-ogy requires better quality, and more accuracy in the description of single ideas, rather than the building up of a great number of new terms.

Yu. A. Zhemchuzhnikov (1948) devoted a special report to the analysis of the facies concept which he considers to be lithologic in character.

He compares concepts of "facies" and of "rocks." Thus, he writes: "of course, there is a difference just as profound, though not always perceptible, between the rock with all it properties and the facies of a deposit " (p. 54). The vagueness of the content of this citation can be seen from the fact that here the rock is compared not with the deposit, but with sedi-mentary facies. On the other hand, the author's definition of "the rock" is not clear. Thus, he writes: "the rock is the combination of all its different primary and secondary features." In this instance the very essence of the subject is lost. This is like saying "the apple is the com-bination of all its characteristics." The author's

words that "the layer and the rock are not the same thing, " are not clear either. The layer or stratum refers to one of the forms of occurrence of sedimentary rocks in the earth's crust.

Yu. A. Zhemchuzhnikov correctly points out that "amongst the rock characteristics one may find a number indicating the genesis of the primary sediment from which the sedimentary rock has formed" (p. 54). Then he continues with "the combination of these features (i. e. , sediment characteristics - V. M.), interpreted to some degree, is the facies" (p. 55). Then further on the same page he defines facies entirely differently: "Facies should also mean the facies characteristics observed in a rock, and the conditions to which they, in the author's opinion, correspond, and the general picture of the formation of the sediment, if the existing data for its restoration are sufficient. " This definition by its author hardly permits one to detect the concrete meaning of the concept of facies. Here are artificially combined different categories such as characteristics of the rock, causes of these characteristics, and finally, the general picture of sedimentation.

Turning to the ancient facies, Yu. A. Zhemchuzhnikov notes that "the facies is neither a layer nor a rock in the sense that it can be felt and held in the hand, but is some representation or concept which is perhaps not ancient either" (p. 56). In summarizing his ideas of the ancient facies, he writes: "we understand facies to be the combination of the sediment characteristics and the conditions of their formation" (p. 57). We cannot consider such a definition of the facies concept very desirable because it leads to indefinite, subjective ideas. We agree with Yu. A. Zhemchuzhnikov that "comparing facies in their alternation, a comparison of facies complexes clarifies not only the conditions prevailing in the area of sedimentation, but also to some degree the entire environment of the landscape" (p. 57). However, this deduction by the author does not conform to those definitions which he applied to the facies concept.

In the opinion of Yu. A. Zhemchuzhnikov the facies concept is " a lithologic or sedimentary-lithologic concept, i. e. , related to the deposit" (p. 58). Facies, as he believes, have three dimensions and "they acquire a more tri-dimensional, physical character in paleogeography. " Thus, Yu. A. , Zhemchuzhnikov in his work has given several distinctive concepts of facies, which obviously does not introduce clarity to this concept though some features described in the article by this author are, in our opinion, correct and deserve some attention.

A few words should be added about an article by L. B. Rukhin "Types of sandy facies" published in the "LITOLOGICHESKII SBORNIK" (1948).

The author evidently understands facies as the conditions of sedimentation in a broad sense. Thus, he writes: "sand may occur under various facies conditions from typical continental (lakes, river sections) to comparatively deep-sea and ocean basins" (1948, p. 85).

It is hardly possible to accept the author's statement that "in contrast to the variation represented by sand facies, a majority of other rocks form under much more uniform environments" (p. 85). In the first place sand is only an element in the complex of sediments which formed as the result of a complex depositional process; it is hardly correct from the facies analysis standpoint to consider it separately from other clastics (conglomerates, clays) and various carbonate and chemical sediments, all of which participate in the sedimentation process in a single sedimentary basin.

In describing the position taken by a number of researchers with respect to the influence of the tectonic factor on the character of accumulating deposits in different geotectonic zones, such as the geosynclines, platforms, etc., - L. B. Rukhin attempted to characterize the deposits forming in different geotectonic zones, the geosynclines, border platform zones, and platforms. What the author means by "the facies conditions" is not clear, since from this concept he excludes "the regime of wave motion" (p. 87), and "the dynamic conditions of deposition, " that is, "the characteristics of the deposition medium" (p. 90). The expression "facies deposition" or sedimentation (p. 89) is not clear either.

The author supplies a table in which he gives the types of sandy facies, their characteristics, and their adaptation to the tectonic areas. The author understands the sand facies or the facies of sands as "a definite type of deposit characterized in its entire extent by a uniformity of lithologic and paleontologic characteristics" (p. 91). He writes: "Thus, of the three principal groups of sandy facies the white quartz sands are most widely distributed [areally]" (p. 88).

Let us turn now to a recent work by L. B. Rukhin (1953) in which he describes his idea of facies in more detail. Whereas in the foregoing work he talks about the "sand facies" or "facies of sands", here he expresses an opinion, that "it is impossible to compare the facies with the deposit or the rock" and then continues: "therefore, we ought not to indicate sandy, limestone-dolomitic, iron-ore, and analogous facies" (p. 290). This, however, does not prevent him from stating at the very beginning of his work that facies refer to rock types. Thus, he writes: "The science of sedimentary rocks considers facies as genetic types of ancient deposits" (p. 7). He speaks of three systematic divisions of sedimentary rocks

which in his opinion are relatively simple categories followed by facies, while larger systematic divisions of the sedimentary rocks are formations.

However, the definition of facies by the author reflects a different meaning. Facies, according to L. B. Rukhin, is "the normal complex of the lithologic and paleontologic characteristics of the sediment which reflect its depositional environment. Thus, facies is a material expression of a sedimentary environment."

In our opinion, the material expression of the depositional environment is represented by the deposits characterized by properties which reflect special features of the sedimentation process and the conditions in which the process took place.

In the opinion of L. B. Rukhin "facies is a broader concept than rock, as facies includes not only material features of the rock, but also characteristics of the conditions under which its initial sediment formed." The author displays some discrepancy in defining the term facies and in applying this designation to most types of deposit. As an example, we cite the following. The author writes: "Marine facies are widespread amongst sedimentary beds. They occupy extensive areas; their cross sections are characterized by a considerable quantity of various organic remnants" (1953, p. 293). In this instance the term "facies" refers to rock type.

Many instances of the author's identifying the term "facies" with rocks can be given. For instance, he writes: "The lagoonal group of facies. . . includes deposits formed in areas intermediate between land and sea" (p. 301); and further, "The facies of large fresh-water basins differ from those of marine deposits mainly in the character of the organic remnants . . . " (p. 303), etc.

The author also uses the term "macrofacies", in which, according to him, are unified, genetically similar facies. He separates "macrofacies of river plains into units consisting of the river bed, flood valley, old channel, and other facies" (p. 292). Also are separated "near-shore shallow-water", "deep-water", etc., macrofacies.

In summarizing the above review, we note that L. B. Rukhin does not follow a fixed interpretation of facies and uses the term freely, giving it different meanings.

B. P. Markovskii in his thesis on "The term and concept of facies" (1948) properly points out the absence "in theoretical geologic literature of sufficiently complete and clear definitions of the facies concept to meet the modern level of geologic knowledge and to avoid an arbitrary treatment of this concept." He cites a number of definitions of the facies concept by different researchers and gives his own definition of it. According to B. P. Markovskii, facies is a surface area of the earth's crust with a definite complex of physico-geographic conditions, which identify both inorganic and organic processes in this given section of the earth's surface during a given interval of time." This definition is similar to that of D. V. Nalivkin of recent facies, and differs from it only by the introduced time element. This situation evidently was brought about by the author's desire to avoid a dual interpretation of the facies concept such as took place in D. V. Nalivkin's work (the modern facies and ancient facies).

A definition of this type, however, leaves vague a determination of the size of a facies; thus, sections of the earth's surface, for whatever section of the crust we may consider, will be "characterized by a complex of physicogeographical conditions which determine inorganic as well as organic processes at a given place and time." It is not clear, either, how such a definition can be in accord with the author's assertion that "facies in geology is a paleogeographic and not a petrographic concept." The author does not consider appropriate the expression: "Deposits of the Middle Devonian are represented by different facies." In our opinion, the quoted expression, to the contrary, is quite correct and is in line with the idea of facies which has been shared, according to the literature, by most researchers.

Examples quoted by the author as correct, such as, "The facies of the Lower Jurassic delta," "the facies of the lithoral zone of the Tertiary Sea," are expressions that do not conform to his definitions of the facies concept. Thus, though B. P. Markovskii defines facies as "a section of the earth's crust. . . ," the term facies in these examples can be understood only as referring to a sediment; otherwise this term loses its meaning and can be dropped.

A few words will be stated concerning the thesis in a paper by V. K. Vassilenko, "Basic concepts of lithology."

According to V. K. Vassilenko, "the joining of separate rock types in the lithosphere takes place in two directions, horizontally and vertically." Here he visualizes that "synchronous kinds of rocks in relation to each other are facies of a stratigraphic horizon" (1948, p. 49), while "the rocks arranged one above another" are considered by him as a "succession." In his opinion, the former reflects the chronological pattern of sediment accumulation and the latter reflects the chronological pattern, indicating only specific relationships of the rocks.

However, V. K. Vassilenko states that

"neither a facies nor a succession may be considered a rock" (p. 49). He believes that use of the word facies instead of rock is not necessary, and that the facies concept as an environmental unit must be abandoned. The meaning given in the thesis by the author to the terms "facies" and "succession" is not clear. It appears then, that the facies of one stratigraphic horizon in relation to the facies of the overlying or underlying horizon must be renamed succession; in other words, the same unit may be considered as a facies and as part of a succession, which is hardly useful.

In her work (1948) M. V. Klenova, citing definitions of the geologic facies by D. V. Nalivkin and of the geochemical facies by L. V. Pustovalov, notes that "in substance there is no difference between concepts of geological and geochemical facies."

M. V. Klenova introduces the term "marine facies", which in her opinion must be applied to a section of the sea bottom characterized by uniform physico-geographical conditions, and has formed during the process of developing a basin or reservoir with uniform composition of the flora and fauna. In order to have two facies actually the same, it is necessary that two sections of the sea bottom experienced an identical history. She writes: "In marine geology we understand facies to be sections of the sea bottom characterized by uniform physico-chemical and biochemical conditions, and having a common source of sediments; that is, having the same genesis for organic as well as mineralogic particles [and] having a uniform flora and fauna which have experienced the same geologic history" (p. 187).

In both definitions of marine facies by M. V. Klenova, a section of the sea bottom is mentioned though the characteristics of it are different. In the first case, it is characterized by physico-geographic conditions having the same floral and faunal composition; in the second case, by identical physico-chemical and biochemical conditions. It is not clear how sections of the sea bottom can have one and the same source of supply, or "the same genesis of their organic as well as mineralogic particles." What is this "uniform flora and fauna having experienced the same geologic history?"

Both descriptions of facies are vague.

The author's assertion that "In the near-shore area each biocoenoses corresponds to one or several facies" (p. 187) also contradicts the definitions of the facies concept.

The author writes that "the biocoenoses of rocks is equal in value to the rock facies, the biocoenoses of sand corresponds to the facies of the shore and marine sand" (p. 188). If in the above sentence the author's definition "a section of the sea bottom" were substituted for the term facies, this would hardly make sense.

If the same thing were done with many other quotations from the author's paper, the picture would be the same. For instance, she writes: "farther from the shore in the shelf area a sudden change of facies takes place in the roughness of relief, i. e., on the slopes." And another vague expression: In the shore facies the principle part is played by processes involving the destruction of the shore line and accumulation of course fragmental material" (p. 188). In this case it says that the processes which "destroy" the shore line take place within the facies themselves. To say nothing of that the shore line does not "get destroyed", but changes or moves, it is difficult to catch the meaning of the facies concept in this quotation.

As M. V. Klenova sees it, "the entire earth's surface can be divided with respect to the character of climatic conditions into facies areas, such as polar, tropic, temperate, and desert" (p. 188.) It is not clear at all why these areas are called facies; there is hardly any necessity to use here the the term "facies" as it does not make these physico-geographic categories any clearer. What is the meaning of her other phrase: "the shore facies of the polar areas are mainly characterized by the glacial deposits? Near the shores these deposits are hardly distinguished from the overland moraine accumulations" (p. 188). Here facies are compared with accumulations which does not conform to the author's definitions of facies.

One could quote many passages from the paper of M. V. Klenova which emphasize the inconsistency between her definitions of facies and the discussion of this concept in which concrete examples of the term facies are used, mostly for designating types of lithologic deposit. For instance, the author writes: "The most characteristic facies of the littoral is pure sand free from argillaceous admixtures" (p. 189), or "the sandy facies pinches out," "the rock facies borders directly on the sill," "the facies of the lake dolomite changes to peat" and so on (p. 191).

The mineralogical characteristics of sediments are presented by this author in the table called "facies of sediments."

Thus, the definitions of the facies concept by M. V. Klenova can hardly be considered fortunate; moreover, strictly speaking, they do not conform to the meaning applied by the author to this term in concrete examples.

D. M. Rauzer-Chernousova gave considerable space in her paper to the analysis of the facies concept (1950). She notes (p. 21) the three most clearly formulated concepts of facies in the literature: facies as the totality of lithologic and paleontologic features of deposits, facies

as conditions of sedimentation, and facies as sections of the earth's surface.

In her opinion (p. 28), the first two of the afore-mentioned interpretations of the facies concept correspond to the term as interpreted by A. Gressly, though they refer to a different meaning of this concept: on the one hand, the totality of conditions of sedimentation, and on the other - the totality of lithologic and paleontologic characteristics of the sediments.

As we see it, N. B. Vassoevich is right when he draws a definite line between these two interpretations of facies, although he is reproached by D. M. Rauzer-Chernousova. The following does not explain how that author interprets the meaning of the facies. In criticizing D. V. Nalivkin, Rauzer-Chenousova writes that "the facies is not a rock but an abstract meaning which includes the analysis of the dynamics of conditions of the medium in time and space." In her opinion, "basically, the facies concept is paleoecological and sedimentational" and it may not be transferred to "recent phenomena" (p. 23). Not all these contradictory interpretations of the facies concept, naturally, have been reflected in the definitions which D. M. Rauzer-Chernousova gives in describing this concept. She proposes to separate out three categories of facies; the basic unit -- "the facies"; the largest unit -- "the formation"; and an intermediate one -- "ther lithotope." D. M. Rauzer-Chernousova writes; "we shall understand facies as primary paleontologic, paleoecologic, petrographic, and geochemical properties of a homogeneous stratum, layer, or bedding plane, which indicate the actual conditions of formation. The geologic facies is represented by a stratum or a layer" (p. 24).

Such a definition of facies can hardly be called desirable, apart from the fact that it does not conform even to the opinions of its own author. First of all, what should we understand as primary paleontological and paleoecological properties of the stratum, the layer, or the bedding plane? Something very subjectively abstract is concealed here. The meaning of this definition as related to "the actual conditions of formation" is not clear either. The formation of what? Evidently, it is not that of the sediment since the facies concept, according to the author, cannot be applied to recent occurrences.

The description of the following facies categories also remains vague. To this, D. M. Rauzer-Chernousova writes: "We shall call the lithotope a combination of characteristics of those deposits of the same type which indicate the average conditions of formation of the facies complex" (p. 24).

This form of definition does not make the concept of facies more precise because the meaning of facies is not clear. Here is discussed the combination of features of deposits with an average depositional environment of a complex of facies. According to the author, facies is not a rock. Then, how is it possible to speak about the conditions of facies formation, for the author understands facies also as the conditions of sedimentation. Finally, the definition of the formation is still less helpful. D. M. Rauzer-Chernousova writes: "In a formation the characteristics of the lithotope are still more generalized, and the lithologic and paleontologic characteristics at times give place to the paleogeographic. The interrelation of the depositional character and the geotectonic regimen as well as the regular combination of definite, genetically related sedimentary complexes become the principal and fundamental characteristics of a formation. The formation concept is, in the first place, tectonic. Geologically it is represented by strata" (p. 24, 25).

This quotation can hardly permit comprehension of what the author means by the term formation. It is not clear what the author means by the paleogeographic characteristics, contrasted with those that are the lithologic and paleontologic. The author contradicts himself saying that of the three categories of facies, the first is an abstract paleoecological and sedimentional concept, but the largest unit of facies is a formation--a tectonic concept. In a summary table the author has already defined formation as "a combination of characteristics of a complex of genetically similar sediments that indicate generalized conditions of the formation of strata of similar genesis." The referring of the three categories of facies to "types of facies occurrences" (p.26) is also difficult to understand. According to D. M. Rauzer-Chernousova's opinion "all researchers have agreed that the facies is the lowest unit for all facies concepts," and yet she applies the term "subfacies", separating under this term the types of reefs.

It is difficult to understand why D. M. Rauzer-Chernousova introduces three distinct definitions of the term facies. Apart from the above quoted statement, she furnishes another: "we shall call the facies a combination of local petrographic and paleontologic features of an assemblage of uniform deposits, characteristics of which point out the physico-geographical conditions of their formation" (p. 26).

This definition, though different, does not improve on the first one. In this instance, the author limits facies to the combination of local petrographic and paleontologic characteristics of an assemblage of uniform deposits, except "geochemical features." What then are "local petrographic and paleontologic characteristics?"

Finally in the summary table, page 25,

the third definition of facies is given: "The facies is a combination of the features of sediments, which indicate the actual conditions under which uniform deposits were formed."

Thus the three quoted definitions differ not only in form but also in substance. It is the same with the definition of concepts of "lithotope" and "formation." Such an approach cannot improve the status of the question of the facies concept.

In substance, D. M. Rauzer-Chernousova in this specific case sets off types of sediments and organic formations as facies.

It is possible, for instance, to point out that the author does not differentiate between such expressions as "facies of the Schwagerina age" and "facies of the Schwagerina horizon" (p. 43, 54, etc.). She distinguishes "facies of shallow waters", "facies of hydractinoid reef", "facies of Bryozoan reef", "facies of underwater shoal", "facies of swift currents", etc. These kinds of facies hardly conform to the definitions of the facies concept given by the author.

G. I. Teodorovich understands geologic facies as "a regular complex of petrographic, paleontologic and geochemical features of deposits, which expresses the paleogeographic and geochemical environment of sedimentation and the diagenesis of the sediment" (1950, p. 6, 7). Such a definition of the geologic facies is very close to the one by A. V. Kazakov (1939, p. 33), and its difference from the definitions of other researchers is that, following A. V. Kazakov, he introduced "the geochemical characteristics of deposits" as facies features. With such an interpretation of facies, the geochemical facies of L. V. Pustovalov becomes a particular case of geological facies, just as the carbonate facies" set off by G. I. Teodorovich is properly considered by him a particular case of the geologic facies (p. 5).

A desirable aspect here is that G. I. Teodorovich, in contrast to many other researchers (D. V. Nalivkin, L. V. Pustovalov, and others) gives only one definition of the facies concept, thus removing doubt concerning his meaning.

But, on the other side, G. I. Teodorovich's definition of the facies concept remains vague since it does not sufficiently reflect the meaning of the facies as distinguished by him. Therefore, there is some discrepancy between the characteristics of facies as recognized by him and his general definition of this term. In defining the facies concept, he speaks about "the complex . . . of the features of deposits," whereas the facies distinguished by him are the deposits characterized by some facies features [such as] faunal and floral (by typical examples of the fossil fauna and flora) or petrographic (structural-textural features) and so on.

Thus, G. I. Teodorovich sets off for instance, such facies as "fusuline (detritus-fusuline)", "bank (shoal water) and shelly (gastropod, pelecypod, or brachiopod)", "facies of mixed composition with participation of many organism groups (large brachiopods, foraminifera, and algae) in the form of detritus integral shells", "microgranular nonschistose limestones", etc.

The definition of the facies concept given by G. I. Teodorovich (the complex of characteristics) does not conform to the meaning resulting from the above quoted characteristics in which the rocks characterized by some typical features are assumed as a basis.

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CLASSIFICATION OF METEORITES ACCORDING TO THEIR CHEMICAL COMPOSITION[1]

by A. A. Yavnel

• *translated by Royer and Roger, Inc.* •

ABSTRACT

Consideration of the main principles of classification of meteorites leads to the conclusion that the primary characteristic should be their chemical composition. From this point of view, the most correct classification was formulated by Prior; however, it needs further perfection.

Definite properties were formed in the chemical composition of meteorites of all three classes, this enabled us to divide the classes into six subclasses: calcium-rich achondrites, calcium-poor achondrites, chondrites, siderolites (mesosiderites), lithosiderites (pallasites) and siderites. These differ in their total iron content, indicated in the ratio of silicate to metallic phase. Depending on the principal composition of the phase (the content of FeO in silicates and Ni in metal), the meteorites are divided into five groups, evidently related to the separate generative bodies.

This distribution is of a distinct character, reflects the genetic connections between different types of meteorites, and can serve as a basis for their classification.

Mineralogical and structural properties were taken into account, and a scheme for classification of meteorites according to their chemical-mineralogical composition and structure elaborated..

An interconnection has been found between the quantity of FeO in silicates and the content and composition of nickel-iron in chondrites, mesosiderites, and pallasites; this is called Prior's group law, as it is observed if meteorites of different groups are compared and absent inside the group itself.

It is concluded that the above law characterizes processes that took place before matter broke up into groups; i. e. before the formation of bodies from which meteorites originated. -- Auth.

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Studies of the substance of meteorites, particularly of their chemical and mineral composition and structure, that have been made in past decades have already yielded rich material on the nature of meteorites. A digest of the existing information is necessary to gain a deeper understanding of this subject, which, along with data on cosmogony, will lead to the main goal of understanding conditions surrounding meteorite formation.

An important part in this will be played by a well-substantiated and strictly systematic classification of meteorites according to their chemical and mineral composition and structure. There is no standard meteorite classification at the present time; a number of students use the Rose-Tschermak-Brezina [1] classification, and the later classification by Prior [2] is used in many papers and catalogues. Finally, some Soviet authors classify the meteorites of the U. S. S. R. according to Academician A. N. Zavaritsky's system [3]. All three classifications sometimes fail to reflect the genetically important properties of meteorites.

All of this suggests the need for critical re-evaluation of the present classifications, with the idea of further improving and unifying them.

A few general remarks on the principles behind the classification of meteorites are in order before this task is undertaken. The production of such classifications may be divided into two stages: 1) choice of criteria for dividing all meteorites into groups; and 2) systematization of these groups. Among the criteria adopted in various meteorite classifications are the chemical and mineral composition of the meteorites, their structure, and often individual details of it; i. e. metamorphic phenomena, color, etc. with regard to the systematization, various classifications use different numbers of subdividions, e. g. classes, subclasses, types, etc., all arranged in different sequences and combinations.

An extremely important step in this direction is selection of proper criteria. This determines the basic principle on which the clas-

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[1] Translated from Klassifikatsiya meteoritov po veshchestvennomu sostavu: Meteoritika, Akademiya Nauk SSSR, Komitet po meteoritam, no. 15, 1958, p. 115-135; an abbreviated version of this paper was published in Geokhimiya, no. 2, 1956.

sification is based. In this connection, it must be kept in mind that the various features reflecting the corresponding properties of meteorites have different values: according to these, they should be assigned their proper place in the classification. Classification by various criteria should reflect the successive steps in formation of meteorite substance.

The main stages in this process may be represented schematically: first as a combination of chemical elements and compounds; followed by formation of mineral aggregates; and then, by the combination of different minerals into various structures. Further change in both the chemical and mineral composition, as well as in the structure of meteorite substance may have taken place during the process of its evolution. These phenomena, however, are of secondary significance and do not affect the continuity of the main process.

On these premises, the primary criterion of a meteorite classification should be the chemical composition, followed by the mineral composition, and then, by other features.

It is to be noted that the chemical criterion has played a definite part in every classification, from the beginning of the nineteenth century when meteorites were classified as stone and iron. Since that time, classification of meteorites by their chemical composition has undergone great changes. According to the existing data, all meteorites may be divided into three classes by their increasing average Fe content as expressed by their nickel-iron content: stone, stony-iron, and iron. These classes, in turn, are subdivided into six types whose meaning will be explained below: 1) achondrites rich in Ca; 2) achondrites poor in Ca; 3) chondrites; 4) siderolites (mesosiderites); 5) lithosiderites (pallasites); and 6) siderites.

The next step in classification is systematization of the material. An orderly classification of meteorites should satisfy the following requirements: 1) it should be built on a single basis common to all kinds of meteorites; 2) at the appropriate stages, it should adhere to a single principle in the grouping of meteorites by consistent features.

Now that these general premises have been established, various meteorite classifications may be examined from this point of view.

It is suitable to begin comparative study of individual classifications with that of Rose-Tschermak-Brezina, 1904 [1]; the nineteenth-century classifications are now only of historical interest.

In that classification, meteorites are divided into two classes: stone (including siderolites) and iron (including lithosiderites); they are subdivided in turn into eight subclasses. This is presented diagrammatically in Figure 1 A,

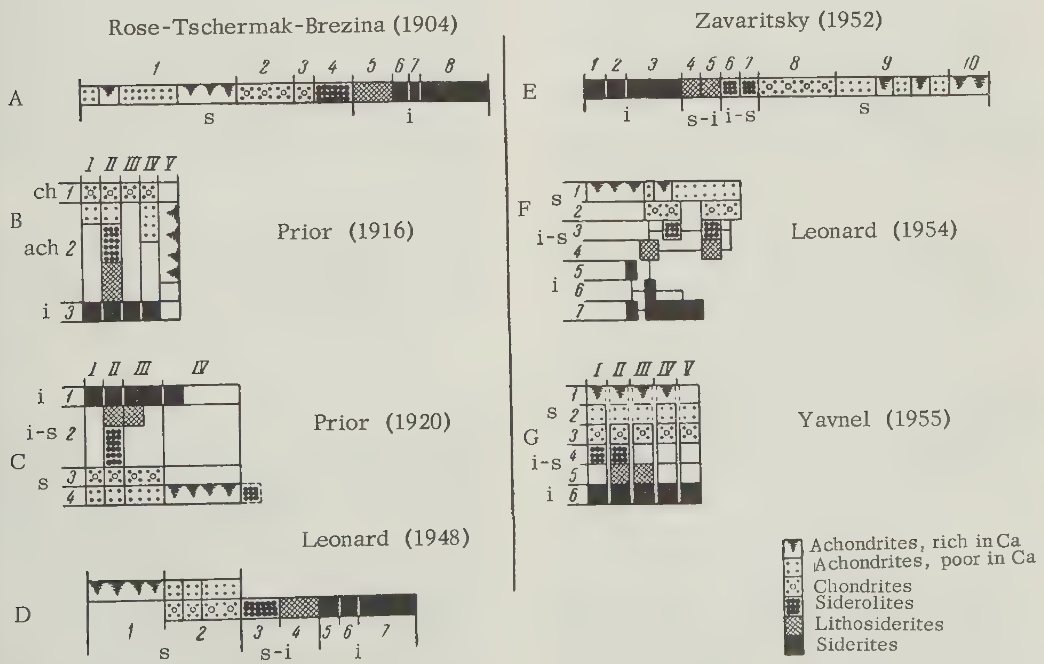

FIGURE 1. Proposed meteorite classifications (i - iron meteorites; s - stone meteorites)

where each kind of meteorite is represented by a rectangle. The position of these rectangles indicates the sequence of transitions from one type to another and their mutual relationships. It appears, for instance, that both types of achondrites belong to the same subclass, whereas iron meteorites are divided into three subclasses. In addition, Brezina divides all subclasses into 78 types; the achondrites are grouped by their mineral composition; the chondrites, by their color and certain structural details; and the iron meteorites, by their structure. Thus, the principle of classification according to a single criterion obviously has not been followed.

In 1907, Farrington [4] proposed a simplified classification for the iron meteorites; these, he divided by their structures and inclusions into 3 subclasses, and, into 12 types obtained by unifying some of the 25 siderite types in the Brezina classification.

Four years later [5], he made another attempt to classify stony meteorites according to their chemical composition, in agreement with the American classification of igneous rocks. These classifications were mutually unrelated, however, and their author eventually lost faith in them [6].

It should be noted here that meteorites, in comparison to terrestrial rocks, have a number of features that must be taken into consideration by any classification. In the variety of minerals and their structural combinations, stony meteorites undoubtedly are different from terrestrial rocks, being much simpler in that respect. In their chemical composition, on the other hand, all meteorites are considerably more diversified. Thus, for example, rocks corresponding in composition to iron and stony-iron meteorites are lacking in the earth's crust; this demands special classification for meteorites.

Thus, it follows that attempts to fit all kinds of meteorites into the framework of terrestrial-rock classification, by drawing analogies between some of them, are doomed to failure.

In 1916, Prior [7] took a step in the right direction when he used chemical composition as the basis of his meteorite classification. He divided all meteorites into three classes, i. e. chondrites, "achondrites" (including siderolites), and iron (fig. 1 B); and, into 23 types. Having established the relationship between the composition of nickel-iron and that of magnesium silicates, Prior (arbitrarily, by his own admission) divided the chondrites into four groups (by the MgO/FeO ratio) and correlated them with other meteorites of corresponding composition, except for those rich in Ca which he separated into a fifth group. The iron meteorites, too, were divided into four groups by their nickel content. This classification by Prior provided the basis of a new systematization of meteorites. Its error lies in that the achondrites are placed between the chondrites and the iron meteorites, breaking the continuity of the transition from one kind of meteorite to another. This was corrected by Prior himself in 1920 [2].

While adhering to his principle in this new classification, Prior introduced certain modifications (fig. 1 C). He divided the meteorites into three different classes: iron, stony-iron, and stone; with the latter subdivided into two subclasses (chondrites and achondrites). A substantial change was the unification of Groups III and IV, for the sake of simplicity, according to the author. Thus, the majority of meteorites were divided into three groups. Here, again, the Ca-rich achondrites and the iron Oktibbeha meteorite were combined in a separate group. Mesosiderites were placed with achondrites in the same group which did not fit the general scheme. This inconsistency was straightened out by Hey in the second edition of the meteorite catalogue [8]. Altogether there are 25 types in this classification. On the whole, it is more orderly than the preceding one. Prior presented his classification in both tabular and linear forms.

This last circumstance inspired Leonard [9] in 1948 to extend the linear form into a chain, while preserving the sequence of the several classes (fig. 1 D). In the Leonard classification as well, meteorites are divided into three classes; stony, stony-iron, and iron; but, with seven subclasses as compared to Prior's four, all of the achondrites are placed in one subclass, but subdivided into two groups, i. e. rich and poor in Ca. In contrast to the Prior system, iron meteorites are divided into three subclasses, according to chemical composition and structure. The subclasses in turn are divided into types, 32 in the later variant of this classification [10]. The only thing left over from the Prior system is the relationship between the low-Ca achondrites and the chondrites. For this reason, the Leonard classification is less orderly and less consistent.

An even greater departure from Prior's classification, that of A. N. Zavaritsky [3], is based on a different principle. This classification, illustrated in Figure 1 E, groups meteorites into four classes (iron, stony-iron, iron-stone, and stony) and 10 subclasses; five of these, in turn, are subdivided into 18 types. As in the Leonard classification, iron meteorites are divided into three subclasses, according to their chemical composition and structure; the chondrites are grouped according to their metamorphism; the achondrites, on the other hand, are not subdivided by their chemical composition, but, classified by their

mineralogical features. For this reason, the previous comments on the Brezina classification also apply to a considerable extent here.

In 1955, a modified classification of meteorites was published by Leonard [11]. He proceeded on the assumption that a meteorite classification should be based on two criteria: structure and mineral composition. Accordingly, he constructed a two-dimensional system (fig. 1 F) similar to that of Prior. He divided all meteorites into three classes (stony, stony-iron, and iron); actually, according to their chemical composition. In addition, the stony and stony-iron meteorites were divided into four subclasses according to their chemical composition; while the iron meteorites were divided into three subclasses according to their structure. The subclasses were subdivided further into 33 types by their mineral composition. Those connections between individual subclasses and types which could not be placed close to each other on the diagram, were indicated by lines.

Inasmuch as mineral composition of the meteorite reflects to a certain extent its chemical composition, excepting purely external criteria for the differentiation of iron meteorites into subclasses, this classification of Leonard's is closer to Prior's than the last one [9].

It can be seen from this brief review that there are differences between the several classifications as to selection of the criteria for classification. In this regard, the Prior classification [2] of meteorites by their chemical composition is the most correct. Furthermore, because of his strict adherence to this single principle, Prior was able to shape his material into a fairly well-proportioned system; this also constitutes an advantage over the others.

Prior's classification will be discussed again here. In the meantime, it must be emphasized that selection of the chemical composition of meteorites as the basis for their classification, makes it imperative to generalize from the analysis results for different types of meteorites.

At the present time, as the author has pointed out [12] certain regular features in the chemical composition of meteorites, it is possible to base the classification of meteorites on strict principles and to introduce into it further refinements.

It is well known that many meteorites, even those of the same type, vary in the content of their principal components. For instance, nickel concentration in iron meteorites ranges from 4 to 60 percent; also, wide fluctuations are observed in FeO and nickel-iron content of the chondrites, etc.

It was believed until recently that, inasmuch as variations in composition of these meteorites is continuous, their differentiation into groups is of necessity arbitrary. As a matter of fact, the situation is somewhat different.

Although it is possible to find meteorites of the same type that form a continuous series with a gradual change in composition, their distribution according to some of their components suggests the existence among them of individual groups with different chemical compositions.

This is illustrated graphically by histograms of meteorite composition: Concentration of typical components is plotted on the horizontal axis, and the number of meteorites of given composition within a definite concentration interval is plotted along the vertical axis.

For this purpose, the six previously named types of meteorites are considered, for convenience, reverse order.

Iron meteorites-siderites, as was pointed out earlier, vary greatly in their nickel content. A summary of published results of the determination of nickel concentrations is found in the Prior and Hey catalogue [8]. By supplementing this with data from more recent publications, this writer has succeeded in gathering 478 analyses of 377 iron meteorites. Without being too selective, he has merely given greater weight to the later of several analyses of the same meteorite.

The entire concentration interval of nickel was broken into segments of 0. 2 percent each; the numbers of meteorites carrying these amounts of nickel were plotted on the graph. The result is presented in Figure 2 which shows the complex distribution of iron meteorites according to their nickel content.

First of all, the graph shows two maxima: one is very conspicuous at 5. 6 percent Ni, and a twin maximum at about 8 percent Ni; these indicate the existence of two meteorite groups of corresponding composition. The meteorites of these two groups together, account for over 80 percent of all of the iron meteorites studied.

A third, and smaller, group of meteorites (approximately 15 percent of the total) is indicated for average nickel content of about 14 percent.

In addition, there are seven meteorites about 5 percent of the total) that contain over 20 percent Ni. Five of these, i. e. the Freda, Wedderburn, San Cristobal, Lime Creek, and Santa Catharina meteorites contain up to 35 percent Ni. The other two, i. e. the Lafayette and Oktibbeha County meteorites with about 60

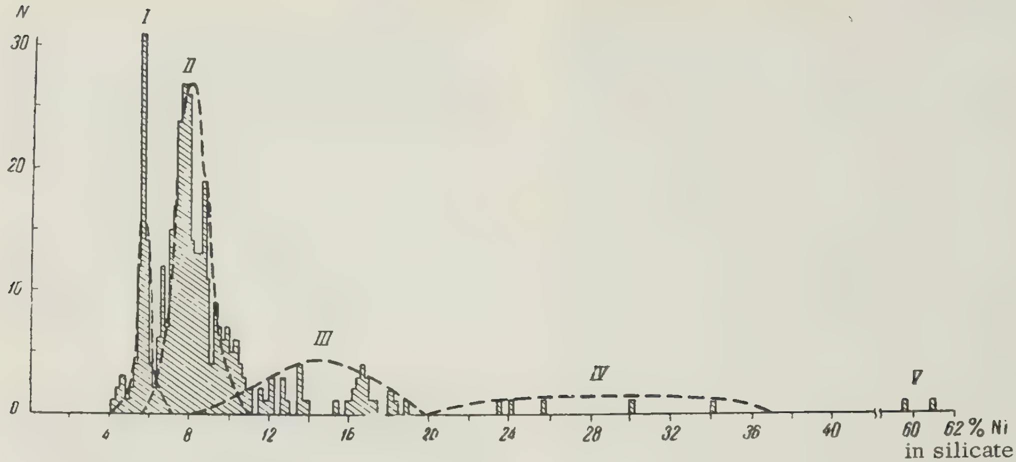

FIGURE 2. Distribution of iron meteorites according to their nickel content

percent Ni content, are at a considerable distance from them.

Thus, to the first approximation, iron meteorites may be divided into five groups according to their Ni content. The nickel-concentration intervals for each group may be estimated from the shapes of the curves comprising the histograms.[2]

The first meteorite group has nickel content in the 4 to 7 percent range; the second, 5.5 to 11 percent; the third, 8 to 20 percent; the fourth, approximately 20 to 40 percent; and the fifth, over 40 percent. These groups are of different widths, increasing with nickel content; some of them mutually overlap.

It should be kept in mind that on the meteorite-distribution curve, the position of maxima corresponding to the known nickel content bears no relation to the diagram for the state of the Fe-Ni system, which determines the phase (mineral) composition and the structure of the iron meteorites. Therefore, coincidence between the groups thus determined and the individual structural types of these meteorites is hardly to be expected.

After similar distribution-frequency graphs for the individual structural types had been constructed, the nickel-concentration ranges from which the type of structure within each group could be seen, were determined for each of them (hexahedrites, octahedrites, etc.). It turned out that the first group consisted principally of hexahedrites and nickel-poor ataxites. In addition, this included most of the very coarse-textured, and some of the less coarse-textured octahedrites. The second group comprised principally the medium-textured and most of the coarse-and fine-textured octahedrites. The third group consisted of very fine-textured octahedrites and nickel-rich ataxites; while the fourth and fifth groups included only the nickel-rich ataxites.

In 1947, Brown and Patterson [13] published histograms showing the frequency of distribution for iron-meteorite composition. Their graph, constructed from 215 analysis results for iron meteorites with nickel content up to 17 percent, also shows two maxima, corresponding to the first two groups above.

In turning to other types of meteorites that consist chiefly of two, i.e. iron and silicate, phases, it should be kept in mind that their chemical composition is determined generally by the composition of these phases and by their relationship. Therefore, at least three magnitudes are necessary in order to characterize these meteorites: two for the composition of each phase, and the third showing the content of one of these in the meteorite.[3]

The characteristic features chosen were: 1) FeO content in the silicate; 2) Ni content in the nickel-iron; and 3) iron-phase content in the meteorite.

The next category of meteorites, lithosiderites, is made up almost entirely of pallasites. The limited number of pallasites, some of

[2] Here, as elsewhere in this paper, no attempt was made to achieve full correspondence between areas under the curves and number of meteorites in the group.

[3] For brevity, the silicate phase (part) of a meteorite will be called the "silicate," and the metallic phase, the "metal."

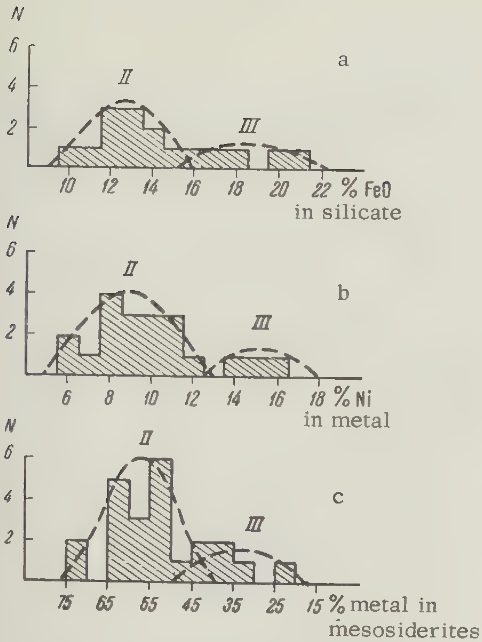

FIGURE 3. Distribution of pallasites by their FeO, Ni, and metal content

them not yet analyzed, precludes definite statistical processing of the data on their composition. Some preliminary conclusions, however, can be made from the material at hand.

Altogether, published data on 23 pallasite analyses were used.

The distribution of pallasites according to FeO content in the olivine (by 1 percent intervals) is illustrated by a graph (fig. 3 a) that shows the olivine of most pallasites to contain 10 to 15 percent FeO, with a 16 to 31 percent content in the remainder. If the pallasites are divided by this feature into two groups, the nickel-iron of pallasites with the smaller amount of FeO will contain 5. 5 to 12 percent Ni; this corresponds to Group II of the iron meteorites (fig. 3 b). The other group of pallasites (containing more FeO) is closer to the iron meteorites of Group III, with its 13 to 17 percent Ni content.

In addition, pallasites of Group II contain 40 to 75 percent nickel-iron (average 57 percent), more than the Group III pallasites of the Itzawisis type, which contain 25 to 45 percent (average about 35 percent), as shown in Figure 3 c. The nickel-iron content is plotted on this graph in decreasing order, by intervals of 5 percent each.

It follows that the pallasites, too, exhibit a regular relationship between the amount and composition of nickel-iron, on the one hand; and the FeO content in the silicate, on the other, as was found in the case of the chondrites by Prior [7].

Thus, the pallasites of the two groups differ both in the composition of their component phases (olivine and nickel-iron) and in their relative amounts.

It may be said in passing that a glance at the graph (fig. 3 c) shows the pallasites of group II, in turn, to be divisible into two subgroups with maxima at about 52 and 62 percent, according to their nickel-iron content. The first group includes, among others the Pallas Iron meteorite; the second, the Brahin meteorite. In addition, these pallasites are known to differ from each other with respect to the shape of their olivine grains; this suggests a difference in the conditions of their formation. Such differentiation, however, within a single group having comparatively homogeneous-phase composition, is analagous to differentiation of an iron meteorite group according to structural features. As such, it is differentiation on a lower level than the one above.

The pallasites are followed by the siderolites, which include the small group of mesosiderites.

The published analysis results for nine mesosiderites are shown on graphs (fig. 4);

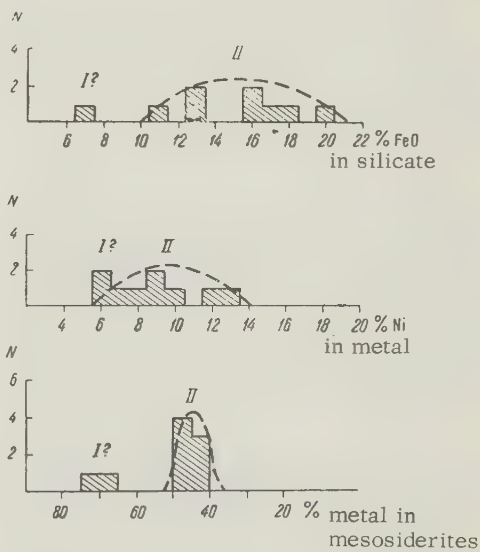

FIGURE 4. Distribution of mesosiderites by their FeO, Ni, and metal content

the FeO, Ni, and nickel-iron contents are plotted at 1-percent intervals.

The scantness of data and the uneven content of SiO_2, MgO, FeO, Al_2O_3 and CaO in the silicate fraction of mesosiderites, however, preclude any definite conclusions on

distribution of these meteorites by their chemical composition.

At the same time, in their composition and especially in the amount of nickel-iron, the mesosiderites appear to constitute an essenially unified group. Judging by the nickel content of their metal fraction, they correspond most closely to the iron meteorites in Group II. It is possible too, that the Bencubbin mesosiderite, which differs from the others in its lower FeO content (7 percent) in the silicate, and in Ni content (5.8 percent) in the metal, along with a high nickel-iron content (70 percent; another rule of Prior?), belongs rather to Group I.

The bulk of the mesosiderites contains 40 to 50 percent nickel-iron; i.e., less than the corresponding Group II of pallasites.

Other types of mixed meteorites-siderophyres (Steinbach) and lodranites (Lodran), according to their composition and structure, may be assigned to an intermediate group between the pallasites and mesosiderites. Thus, the siderophyres are very similar to mesosiderites in chemical composition but have pallasitic texture, whereas the lodranites, with their composition intermediate between pallisites and mesosiderites, exhibit fine texture similar to that of pallasites.

A separate position is occupied by the grahamites, also represented by a single meteorite (Nechayevo). With respect to its content of silicate and nickel-iron grains, this meteorite is close to the Group IV meteorites, whereas its metal fraction contains less nickel.

Next are the chondrites, which make up the bulk of all stony meteorites. Despite their comparatively large number, however, adequate data are lacking on their chemical composition; because, most of the analyses are erroneous in one way or another. Urey and Craig [14] succeeded, after painstaking effort, in selecting 93 of the most reliable results[4] to which the present author has added 6 of the latest analyses.

Thus, data on 99 chondrites were used in the preparation of this paper.

Distribution-frequency histograms for chondrites are presented in Figure 5, where the content of all components are plotted at 1-percent intervals.

According to these graphs, chondrites may be divided into four groups by their chemical composition. Group I contains 5; Group II, 39; Group III, 50; and Group IV, 5 chondrites. They are best differentiated by the amounts of their nickel-iron; the corresponding graph exhibits four maxima, i.e. at 25.5, 18.5, 8.5, and 3 percent. Similar graphs by Brown and Patterson [13] for 184 stony meteorites, also show individual maxima close to these. On their graph, the last maximum (0 to 3 percent metal) is considerably higher because those authors included analyses not only of chondrites but of achondrites as well. The latter are distinguished by their low nickel-iron content.

If the distribution curves for chondrites, according to FeO content in the silicate as determined for each of these four groups, are plotted on a general graph, the picture will be like that in Figure 5 A, with maxima at about 1, 14, 17 and 25 percent FeO.

The distribution curve for chondrites, according to the nickel content in their nickel-iron, is obtained in a similar manner; the maxima, or average values, are at 6.5, 9, 14 and 30 percent Ni.

Comparison of this curve with that showing iron-meteorite composition (fig. 2) also reveals certain correspondence of these chondrite groups to the first four iron-meteorite groups. A correlation of all three graphs will be given presently. It may be stated here that these graphs make possible a positive differentiation of all chondrites by groups. For instance, Group I, besides differing in the amount of metal, differs sharply from the others in FeO content; whereas Group IV stands out for its Ni concentration in nickel-iron.

The groups thus determined have turned out to be close to the four chondrite groups, i.e. those of the Hvittis, Cronstad, Baroti, and Soko-Banja types in the first classification by Prior [7].

On the other hand, Wahl [15] also has subdivided the chondrites into four type groups; in his opinion, one of them represents the white-chondrite type and the three other types that differ from it in their FeO and nickel-iron content. Here too, there is great similarity between these chondrite groups and those distinguished by the present author.

Finally, Urey and Craig [14] discovered two chondrite groups differing from each other in their total Fe content. In terms of the previous data, chondrites with high Fe content (group "H" in Urey and Craig) include the first two groups, while chondrites with low Fe

[4]Urey and Craig cite data on 94 chondrites; one of which, however (Vavilovka) belongs to the achondrites.

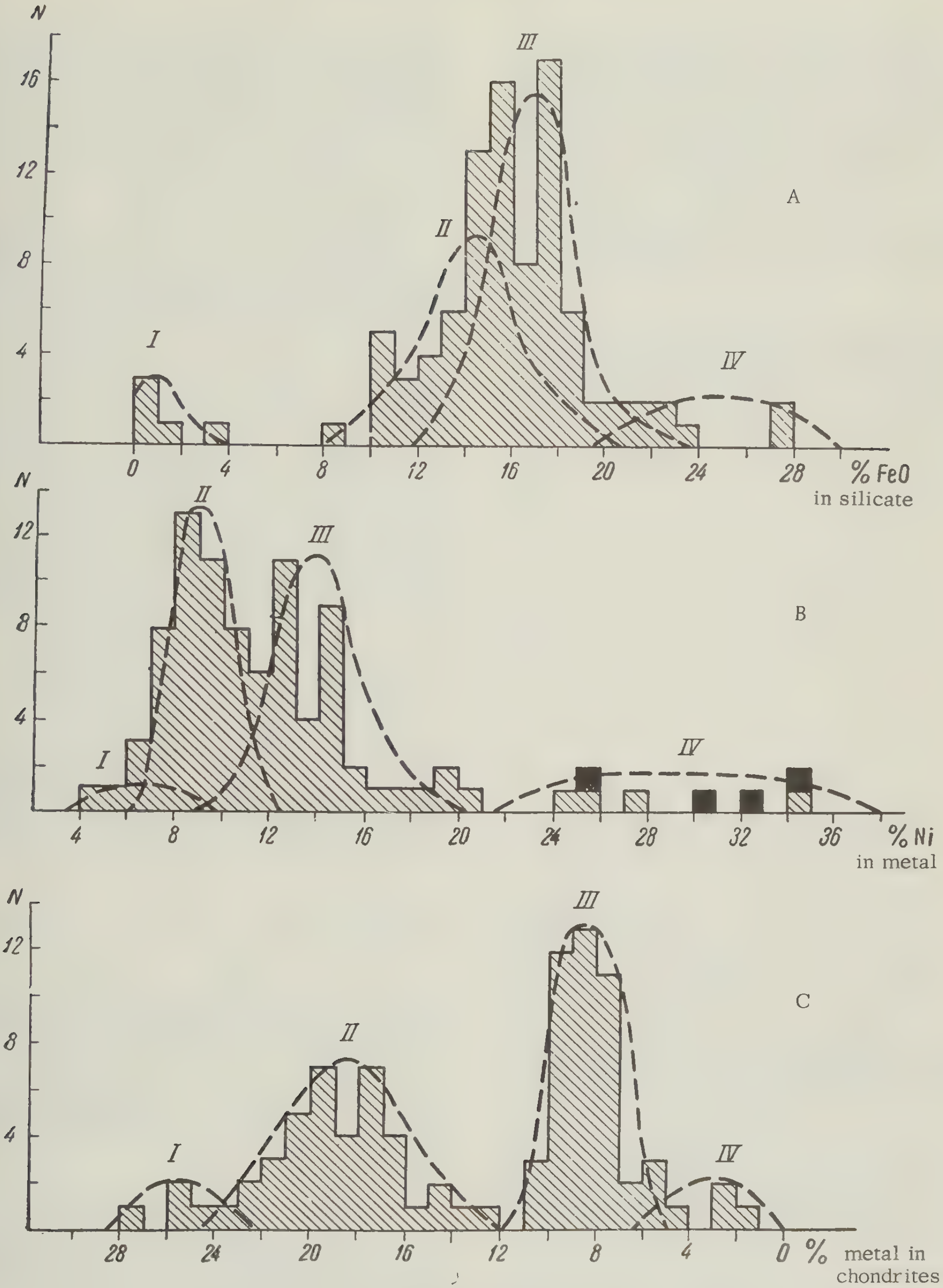

FIGURE 5. Distribution of chondrites by their FeO, Ni, and metal content
FIGURE 5

content (group "L") include the other two. In addition, Urey and Craig distinguished two small groups, /H/ and /L/, that turned out to correspond to the author's Groups I and IV. Thus, according to these two authors as well, chondrites may be divided into four groups.

All of these results confirm the author's conclusions on differentiation of the chondrites into four groups.

In these groups, the most conspicuous of the regular compositional features (traceable on

the graphs) is the definite continuity of change in the amounts of FeO, Ni, and nickel-iron in the individual groups. Specifically, the transition from one chondrite group to another takes the form of an increase in average FeO content in the silicate, in Ni content in the metal, and in decrease of the metal phase.

As mentioned previously, the relationship between these two auantities in chondrites was first discovered by Prior, and is known as "Prior's Law," or, rather, Prior's Rule.

This may be illustrated by a graph in which the percentage of nickel-iron in chondrites, and, of nickel in metal, are plotted along its axes (fig. 6). These are the most indicative properties.

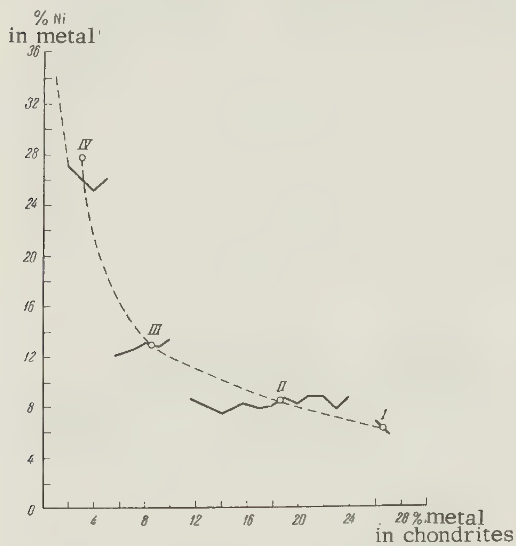

FIGURE 6. Nickel-iron content as a function of the amount of metal in chondrites

By connecting the points of average magnitudes of the Fe-Ni and Ni percentage content in different chondrite groups with a smooth curve, one may obtain a general expression for their relationship. If, on the other hand, average Ni content in the metal of individual chondrites that differ in their nickel-iron content is plotted on the same graph, the single smooth curve will be replaced by four, roughly horizontal steps that correspond to individual chondrite groups.

The conclusion is that Prior's Rule is valid only for chondrites as a whole, in the comparisone of their individual groups. It does not hold within a group, because the nickel-iron composition does not depend on the amount of metal in chondrites of the group.

It follows, then, that doubts voiced by many authors [14] regarding the validity of Prior's Rule are to a certain extent justified. Another regular feature emerges; however, this is of a discrete, rather than continuous, nature contrary to Prior's thesis [7]; and, is related to the existence of definite meteorite groups that have different chemical compositions. This regular feature, which may be called the Prior's "Group" Rule, has been observed not only among the chondrites but among the pallasites and mesosiderites as well.

On the basis of these data, it may be assumed that the original meteorite substance passed through at least two stages during its evolution. The first stage was characterized by differentiation of matter, prior to emergence of the groups; and, was accompanied by qualitative changes in the phase composition associated with changes in their quantitative ratios.

This process is sometimes explained by the change in some of the metallic iron to FeO, depending on the amount of residual oxygen. This would have led to change in amount of silicate phase at the expense of metal; to variation of FeO content in the silicate; and, to comparatively greater Ni concentration in the metal. Besides the oxygen, the difference in total Fe content of the individual groups also was important. Thus, the Fe content in the chondrites of Group I is 29.4 percent; in Group II, 28.7 percent; in Group III, 22.4 percent; and, in Group IV, 21.7 percent. These figures show the decrease in total iron from Group I to Group IV; whereas, the amount of FeO increases in the same sequence. This divergence emphasizes the difference in composition and ratio of the phases.

At this stage, these regularities in the differentiation of matter according to composition reflect Prior's Law."

The second stage took place after the differentiation of meteorites into groups. Although some differentiation of matter continued within each group, average composition of the phases was determined no longer by their number, as before, despite considerable fluctuations in both of these magnitudes. As a result, Prior's Rule does not operate within the individual groups.

This suggests a substantial difference in conditions of the process, in the two instances; and, possibly, different phases of meteorite substance during the first and second stages.

If each meteorite group were associated with an individual planet, as has been suggested before [12], it may be assumed that the first stage; i. e. prior to the differentiation into groups, corresponded to the period preceding formation of the planet. Thus, Prior's Group Rule should reflect processes in the protoplanetary substance when it existed in a gaseous dust state that promoted more vigorous inter-

action of chemical elements and their compounds.

To return to the composition of the chondrites: These include a small group of meteorites that are of somewhat different composition; i. e. the carbonaceous chondrites.

According to their composition, they may be divided, roughly, into three types: Orgueil, Migei and Felix.[5] The differences between carbonaceous chondrites are clear from Boato's data [17] on their isotopic composition of hydrogen and carbon, as well as from their content of sodium and potassium, as determined by Edwards [16].

In their nickel content in nickel-iron (25 to 35 percent), the Migei-type meteorites (marked with black squares in Figure 5 (B), apparently correspond to the chondrites of Group IV. According to petrographic data, however, the actual nickel-iron content in carbonaceous chondrites is no more than 2 percent. Consequently, the relative concentration of nickel in the metal does not exceed 50 percent. This makes it possible to assign the carbonaceous chondrites to Group V.

The next type of meteorite, nearest the chondrites, is that of the calcium-poor achondrites. Data on chemical composition of 17 achondrites of this type, taken mostly from a summary by Urey and Craig [14], are shown on graphs in Figure 7. In order to present a full picture of their composition with small admixtures of metal, the content of all principal components of their silicate fraction is

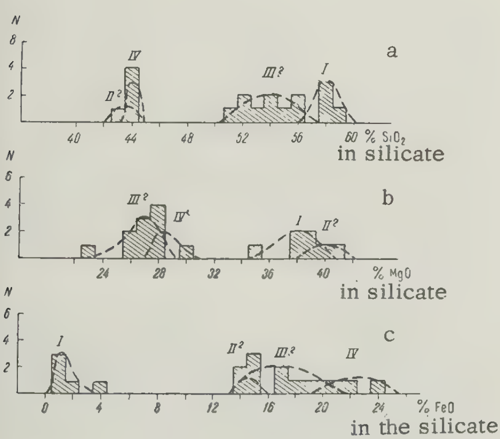

FIGURE 7. Distribution of calcium-poor achondrites by their content of SiO_2, MGO, and FeO

represented graphically: SiO (fig. 7 a); MgO (fig. 7 b); and FeO (fig. 7 c).

From these data, the calcium-poor achondrites may be divided into four groups with average silicate compositions in percent (and increasing FeO content) as follows:

	I	II	III	IV
SiO	58	44	53	44
MgO	38	40	27	29
FeO	1	14	17	28

Group I includes the enstatite achondrites: aubrites[6]. Group II consists of the ureilites which contain a small but significant admixture of carbon. Group III comprises two types of achondrites, i. e. diogenites and rodites, with the same chemical composition. It may be noted in passing that Wahl [18] also pointed out the similarity in composition of the Roda achondrite to that of diogenites. It follows that rodites should be assigned not to the amphoterite group, as has been done in the Prior classification [2], but to that of the diogenites. Group IV includes amphoterites; and Group V, apparently, chassignites.

The great similarity in composition of amphoterites and chondrites of Group IV is conspicuous.

The nickel content in nickel-iron is known for a few achondrites only. The composition of nickel-iron in aubrites is better known; here, average nickel content for three meteorites (excluding the Bishopville meteorite) is 5. 6 percent, and corresponds to that of iron meteorites in Group I. The nickel content in amphoterites (including three achondrites is 27 to 33 percent, corresponding to that of the Group IV iron meteorites. There are no reliable data on nickel-iron composition of ureilites and diogenites; thus, they cannot be assigned to any group by this criterion. They are intermediate between Groups I and IV, according to their FeO content.

Data on the amount of nickel-iron in achondrites do not reveal conclusively any consistent change from one achondrite group to another. On the basis of a very limited number of analyses, it may be considered that this content is 0. 2 to 9 percent in aubrites, 5 to 9 percent in ureylites, up to 1 percent in diogenites, and about 3 percent in amphoterites.

Thus the existing data on FeO content in the silicate, of Ni in the metal, and of nickel-iron

[5] A similar differentiation of carbonaceous chondrites by their composition apparently has been made by W Wiik; this is mentioned by Edwards [16].

[6] Here, as elsewhere, the names of achondrite types are taken from Prior [2], unless otherwise stated.

in calcium-poor achondrites, lead to the tentative conclusion that Prior's Rule does not apply in this case. Further study on composition of this type of meteorite will produce a more definite answer.

Should this conclusion be confirmed, possible cause of such lack of regularity might lie in the considerable secondary alterations in achondrite composition, which may have camouflaged the original regular features of their composition.

Finally, the last type of meteorites are calcium-rich achondrites, which contain a very small amounts of nickel-iron (up to 1 percent).

As in the preceding instance, their composition must be characterized by content of principal components in the silicate fraction: i. e. SiO_2 (fig. 8 a); MgO (fig. 8 b); FeO (fig. 8c); Al_2O_3 (fig. 8 d); and CaO (fig. 8 e). These data from Urey and Craig [14] on composition of 25 achondrites of this type, show that calcium-rich achondrites fall into several groups differing substantially in their basic chemical compositions.

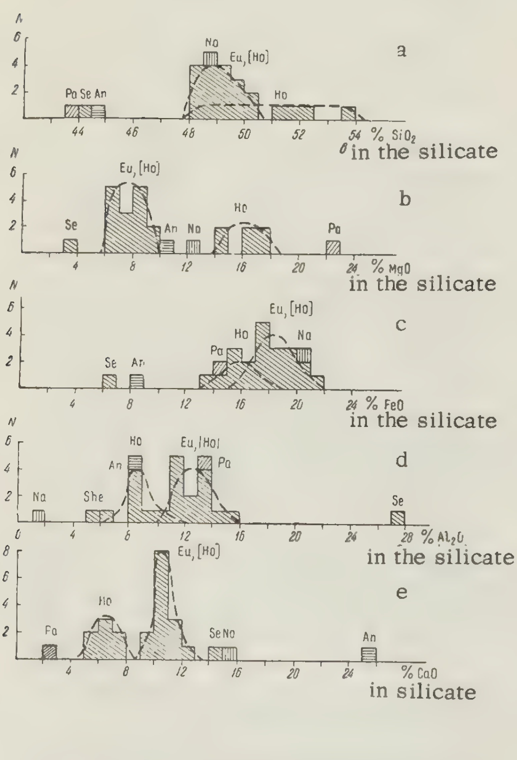

FIGURE 8. Distribution of calcium-rich achondrites according to their content of SiO_2, MgO, FeO, Al_2O_3 and CaO

Eu - eucrites; She - shergottites;
Ho - howardites; An - angrites;
Na - nakhlites; Ho - howardite eucrites;
Se - Serra de Mage; and Pa - Pampa del Infierno.

Proper sequence of the groups is hard to determine in this instance because of the lack of data on nickel-iron composition. The groups will be described, therefore, in the order of their distribution.

The largest group has the following average silicate composition: MgO, 8 percent; FeO, 18 percent; Al_2O_3, 13 percent; CaO, 10 percent. This group includes most eucrites and some howardites (such as the Luotolax, Pasamonte, Petersburg meteorites). The Shergotty shergottite, whihc differs only in having lower Al_2O_3 content (5.9 percent), apparently belongs here as well.

The next group, consisting chiefly of howardites (and the Binda and Chaves eucrites), contains 51 percent SiO_2, 16 percent MgO, 16 percent FeO, 9 percent Al_2O_3, and 7 percent CaO.

It may be seen from these data that eucrites and howardites really belong to two groups of different compositions: ophitic (eucrite) texture predominates in one; and fine-clastic (howardite) texture in the other.

The other groups of calcium-rich meteorites are represented by single specimens: the Angra dos Reis angrite with silicate composition (in percent) of SiO_2, 45; MgO, 10; FeO, 9; Al_2O_3, 9; and CaO, 25; the Nakhla nakhlite with SiO_2, 49; MgO, 12; FeO, 21; Al_2O_3, 2; and CaO, 15; and the Serra de Mage eucrite, which stands out from the other eucrites in its composition: SiO_2, 44; MgO, 3; FeO, 7; Al_2O_3, 27; and CaO, 15. The Atsumamura specimen, whose meteoritic origin is doubtful because its composition is similar to that of a typical gabbro (troilite?), has been omitted.

In addition, there is the well-known Pampa del Infierno howardite, with its peculiar composition (SiO_2, 43 percent; MgO, 22 percent; FeO, 14 percent; Al_2O_3, 13 percent; and CaO, 3 percent;silicate). This meteorite differs from the other calcium-rich achondrites in its high nickel-iron content and low CaO content, which places it intermediate between these and the calcium-poor achondrites. In its Ni content in the metal (about 12 percent), this meteorite corresponds to the Group II or III iron meteorites.

With regard to all other calcium-rich achondrites, comparison of their individual groups with other meteorites calls for data on composition of their nickel-iron content. It should be noted, at the same time, that existing analyses of eucrites and howardites show mere traces of nickel in samples containing tenths of one percent of iron.[7]

[7]Nickel-iron of the Zhmeni howardite was identified erroneously as Fe_2Ni; this formula was used for the Ni computation (0.11 percent) cited by Urey and Craig [14].

This last circumstance suggests that these meteorites cannot belong to the group of the iron Oktibbeha meteorite (as Prior believed[2]), which contains more nickel than iron. They correspond rather to the first groups, which contain less nickel. Also, this is confirmed indirectly by the nickel-iron composition in the Pampa del Infierno howardite.

As was mentioned above, calcium-rich achondrites were first isolated by Prior in a separate group attached to the calcium-poor achondrites, on the amphoterite side; thus, it was essentially a continuation of this class of achondrites.

The data cited, however, do not agree with these conclusions of Prior; and suggest,rather, that calcium-rich achondrites should occupy the same position in relation to calcium-poor achondrites as do the latter in relation to chondrites, i. e. above rather than beside them.

In summarizing this review of the individual types of meteorites, it should be pointed out that there is a definite relationship between them that is best traced by their nickel-iron. A change in the amount of nickel-iron in meteorites signifies a transition from stone to iron meteorites by way of intermediate types. A comparison of the nickel-iron composition in the several types of meteorites (with respect to the content of Ni) emphasizes their genetic relationship.

The relationship between types of meteorites may be illustrated by a graph on which the percentage content of nickel-iron is plotted along the vertical axis and that of nickel in the metal, along the horizontal axis (fig. 9). The points correspond to average values of the quantities plotted along the axes for individual groups of different meteorite types. The maximum scattering of these values is shown by segments. Their probable deviation from the average, however, is substantially less, as shown in the graphs (figs. 3, 4, 5).

Because of the lack of data on nickel-iron composition in ureylites and diogenites, their percentage of Ni has been assumed to correspond tentatively to Groups II and III. For the same reason, calcium-rich achondrites are represented tentatively by a straight line approaching zero content of nickel-iron, and located in the interval of Ni concentration in metal of the first four meteorite groups.

The points corresponding to each type of meteorite are connected by broken lines that illustrate graphically the relationship between the amount and composition of nickel-iron: specifically, the operation of Prior's "Group Rule" in chondrites and, apparently, in pallasites and mesosiderites.

A study of this graph reveals that all six types of meteorites differ quite clearly in their nickel-iron content, apart from other substantial differneces in their composition, as discussed earlier. At the same time, they exhibit a certain periodicity in the composition of their nickel-iron.

All of this suggests a differentiation of the three basic meteorite classes, stone, stony-iron, and iron into the following subclasses:

1) achondrites, rich in calcium;
2) achondrites, poor in calcium;
3) chondrites; 4) siderolites; 5) lithosiderites; and 6) siderites.

The division of meteorites into subclasses should correspond to the periodicity in chemical composition of their nickel-iron. For this reason, iron meteorites cannot be divided into subclasses, as has been done by many authors.

On the other hand, the different meteorite subclasses correspond to definite groups, according to their nickel content in the metal, as was shown earlier [12] and again confirmed in this paper; this may be seen in Figure 9. The correspondence between nickel-iron composition in different subclasses of each group indicates their genetic connection and makes it necessary to consider the group an independent unit. It must be emphasized that this division into both subclasses and groups is not arbitrary, but follows from the observed regular differences in the compositions of meteorites.

A portion of the meteorites in a subclass belonging to one of the groups forms a family of meteorites with similar chemical compositions, i. e. the "chondrites of Group I" or the "siderites of Group III, " etc. In this terminology, the previously named meteorite groups in different subclasses correspond to individual families.

This division of meteorites by their chemical compositions into classes, subclasses, groups, and families, as well as their systematic arrangement, forms the basis for the meteorite classification diagrammatically illustrated in Figure 1(g), so that it may be compared to other classifications.

In its construction, this classification is quite similar to Prior's (fig. 1 c). Its main difference from the latter is that calcium-rich achondrites and mesosiderites have been distinguished as separate subclasses; thus, the number of subclasses increases from four to six and the number of groups, from four to five.

This difference between the two systems results principally from the fact that the

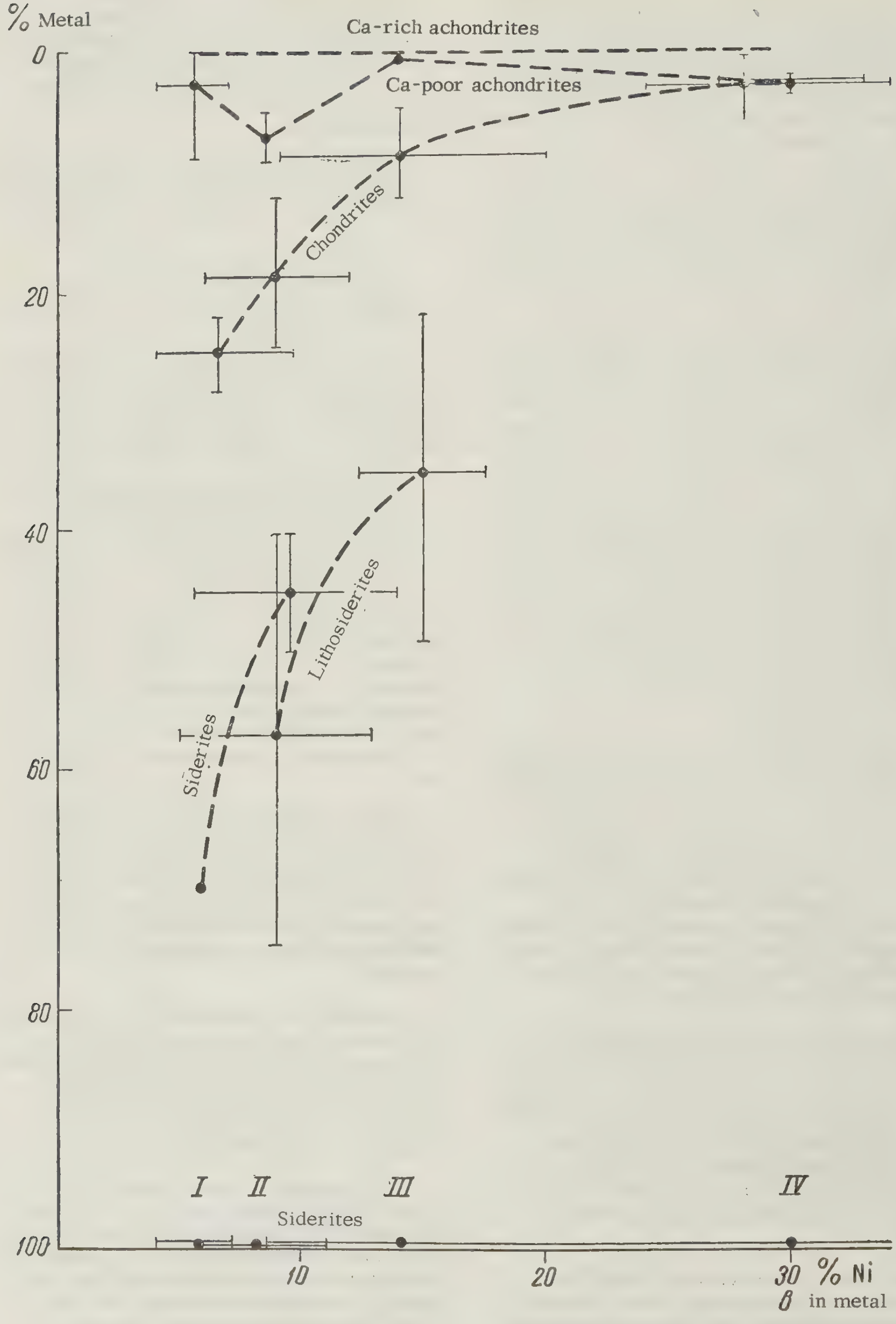

FIGURE 9. Relationship between the Ni content of metal and the amount of metal in meteorites of different types

TABLE 1. Suggested classification of meteorites by their chemical and mineral composition and structures

Class	Subclass	Amount of nickel-iron in percent	Composition of nickel-iron: 4 - 7 % (Group I)				5.5 - 11 % (Group II)				8 - 20 % (Group III)			
			Chemical composition (%)	Minerals	Structure	Type	Chemical composition (%)	Minerals	Structure	Type	Chemical composition (%)	Minerals	Structure	Type
Stony	Achondrites rich in Ca (eucrites, howardites, angrites, nakhlites)	<1	SiO_2 - 49 MgO - 8 FeO - 18 Al_2O_3 - 13 CaO - 10	Clinohypersthene Anorthite (maskelynite)	Achondrite (ophitic) (finely clastic)	Eu (She) Ho	SiO_2 - 51 MgO - 16 FeO - 16 Al_2O_3 - 9 CaO - 7	Hypersthene Clinohypersthene Anorthite	Achondrite (fine-clastic) (ophitic)	Ho Eu	SiO_2 - 43.9 MgO - 10.0 FeO - 8.6 Al_2O_3 - 8.7 CaO - 24.5	Augite	Achondrite	An
	Achondrites poor in Ca	<8	SiO_2 - 55 MgO - 36.5 FeO - 1(0-2) Fe-Ni - 0.2-8	Enstatite diopside Enstatite	Achondrite Achondrite	Au [Bu] Au [Chl]	SiO_2 - 40 MgO - 37 FeO - 13 Fe-Ni - 7 C - 1 - 2	Olivine Clinobronzite Carbonaceous matter	Achondrite	V	SiO_2 - 53 MgO - 26.5 FeO - 17(13-21) Fe-Ni - <1	Hypersthene Olivine	Achondrite brecciated Achondrite	Ro Di
	Chondrites	2-28	SiO_2 - 41.5 MgO - 23.2 FeO - 1(0-3) Fe-Ni - 26(22-28)	Enstatite Kamacite Taenite	Chondrite Crystalline	Cek	SiO_2 - 36 MgO - 23.2 FeO - 11(5-15) Fe-Ni - 18.5(12-25)	Bronzite Olivine Kamacite Taenite	Chondrite Crystalline Spheroidal	Ck Ce, Cg	SiO_2 - 39.5 MgO - 24.8 FeO - 14.2(10-20) Fe-Ni - 8.5(5-12)	Hypersthene Olivine Kamacite Taenite	Chondrite Spheroidal Crystalline Spheroidal	Cck Cc, Cw, Cg
Stony-iron	Siderolites	40-70	SiO_2 - 14 MgO - 10 FeO - 2 Fe-Ni - 70	Kamacite Enstatite ? Olivine	Mesosiderite	M	SiO_2 - 25 MgO - 14 FeO - 8 Fe-Ni - 45(40-50)	Bronzite Olivine Kamacite Taenite	Mesosiderite	M				
	Lithosiderites	25-75					SiO_2 - 17 MgO - 20 FeO - 5.5 Fe-Ni - 57(40-75)	Kamacite Taenite Olivine	Pallasite (rounded olivine grains) (angular olivine grains)	Pk Pr	SiO_2 - 24 MgO - 27 FeO - 12 Fe-Ni - 35(25-50)	Olivine Kamacite Taenite	Pallasite	P
Iron	Siderites	>75	Fe - 93.5 Ni - 5.6 Co - 0.5 P - 0.25	Kamacite Kamacite Taenite	Ataxite Hexahedrite Octahedrite	D_1 H Ogg	Fe - 91 Ni - 8 Co - 0.55 P - 0.2	Kamacite Taenite	Octahedrite	Og Om Of	Fe - 85 Ni - 14 Co - 0.7 P - 0.1	Kamacite Taenite	Octahedrite Ataxite	Off D_2

TABLE 1. Suggested classification of meteorites by their chemical and mineral composition and structures (Concluded)

Class	Subclass	Amount of nickel-iron in percent	Composition of nickel-iron (concluded): 20 - 40 % (Group IV)				< 40 % (Group V)			
			Chemical composition (%)	Minerals	Structure	Type	Chemical composition (%)	Minerals	Structure	Type
Stony	Achondrites rich in Ca (eucrites, howardites, angrites, nakhlites)	<1	SiO_2 - 49.0 MgO - 12.0 FeO - 20.7 Al_2O_3 - 1.7 CaO - 15.2	Diopside Olivine	Achondrite	Na	SiO_2 - 43.4 MgO - 3.2 FeO - 6.6 Al_2O_3 - 27.2 CaO - 14.5		Achondrite	Eu [Se]
	Achondrites poor in Ca	<8	SiO_2 - 40 MgO - 26 FeO - 20(18-22) Fe-Ni - 3	Olivine Hypersthene	Achondrite	Am	SiO_2 - 37 MgO - 34 FeO - 27	Olivine	Achondrite	Cha
	Chondrites	2 - 28	SiO_2 - 39.5 MgO - 25.2 FeO - 20.5(17-26) Fe-Ni - 3 (1- 6)	Olivine Hypersthene	Chondrite Spheroidal	Cc, Cg, Cw	SiO_2 - 29 MgO - 20 FeO - 21 Fe-Ni < 2 C - 2	Olivine (Chlorite) (Graphite) (Amorphous C)	Chondrite (stony)	K
Stony-iron	Siderolites	40 - 70								
	Lithosiderites	25 - 75								
Iron	Siderites	> 75	Fe - 68 Ni - 30 Co - 1 P - 0.1	Taenite Kamacite	Ataxite Acicular	D_2	Fe - 38 Ni - 60 Co - 0.7? P - 0.1	Taenite	Ataxite	D_2

present division into subclasses and groups is founded on actual differences in chemical composition of meteorites and the associated regular features; whereas, Prior had regarded the subdivision boundaries as arbitrary and accordingly had attached no significance to their number.

In carrying out this classification by chemical composition, meteorites should be further differentiated by their mineral composition. Hence, the differences in mineral composition within each family should be the criterion for subdividing each family into types.

By this criterion, for instance, eucrites should be divided into two types: those with maskelynite (shergottites) and those without it. Similarly, aubrites (calcium-poor achondrites of Group I) may be divided into two types: those containing diopside (the bustites of Tscharmak's terminology) and those lacking it (chladnites of the same classification). The carbonaceous chondrites of Group V should be subdivided, in conformity with the Zavaritsky classification, into chloritic and nonchloritic types. The iron meteorites, too, contain meteorite types of different mineral compositions within a single family; e. g. , the siderites of Group I, which may be divided on this basis into those composed of kamacite alone, or, of kamacite and taenite, etc.

The next criterion for continuing the classification of types into subtypes, is structure. Examples of structural differences in meteorites of the same type are the crystalline and spherical chondrites of Groups II and III; the pallasites of Group II with differently shaped olivine grains; the nickel-poor ataxites and hexahedrites of Group I; the octahedrites of Group III with different degrees of refinement in the Widmanstätten pattern; etc.

For better comparison of the meteorites in regard to their chemical and mineral compositions and structures, the classification proposed in this paper is presented conveniently in the following table where all of these criteria are given their proper weight. (table 1).

The column "chemical composition" lists average content of the main components and typical admixtures in meteorites. The "minerals" column lists the principal and secondary minerals . The individual minerals are ranged in order of their distribution in a given meteorite type .

The type and nature of structure are indicated in the column so designated. The "type" column gives the type symbol in the Rose-Tschermak-Brezina-Prior classification.

Calcium-rich achondrites are placed in the upper part of the table; siderites (iron meteorites), in the lower part.

The position of ureilites and diogenites, and that of calcium-rich achondrites, has been indicated tentatively for the reasons stated previously.

At the same time, guided by various considerations, an attempt also has been made to arrange some of the meteorite types in definite sequence. For instance, the average amount of nickel-iron in the crystalline chondrites (Ck) of Group II is somewhat lower than in the spheroidal chondrites (Cc). For this reason, they have been placed above. For the same reason, pallasites of the Pallas Iron type (Pk) have been placed above the Brahin (Pr) type pallasites. Inasmuch as the specific gravity of meteorites increases from the stony to the iron class, siderites of lower specific gravity have been chosen as the boundary types in each iron-meteorites group. These are the siderites with relatively lower nickel content [17]. This also determines their corresponding arrangement according to the structure.

The list of meteorite types in the table does not pretend to be complete; only the most representative types have been selected. This table can,of course, be expanded on the basis of the principles previously indicated. This will give a fuller picture of the change in chemical and mineral composition, structure, and other features of meteorites in proceeding from one type to another within a group.

This becomes even more important if meteorites of each group are assumed to be parts of a single planet. In this event, consecutive distribution of different meteorite types will correspond to a cross-section of the stratified planet from which meteorites of the given group have been formed. Iron meteorites represent the central part of the planet and achondrites, its periphery.

Proceeding from this concept, certain characteristic details become immediately perceptible in the table. Thus, as was noted earlier [12], the metamorphism in meteoritic substance appears to have taken place deep within large bodies. The fact that meteorites with altered structures, such as crystalline chondrites and nickel-poor ataxites, are found above unaltered meteorites of the same composition, suggests that metamorphism in these bodies proceeded from the periphery to the center.

Alteration of the olivine grains in pallasites, possibly another aspect of structural metamorphism, also is of interest. Increase in the amount of olivine with depth is clearly noticeable in the change in mineral composition from mesosiderites to pallasites.

General study of the table also reveals that, because of the division of the meteorites into groups, individual subclasses scarcely overlap in their nickel-iron content. This is especially well illustrated in Figure 9. There are, for instance, chondrites with as much as 28 percent metal, and pallasites with less than 25 percent. Because they are found in different groups, i. e. in different places, it may be assumed that conditions prevalent there caused these differences in their structure.

An examination of individual groups will show that various transitional forms exist between adjoining families. Forms of siderophyres and lodranites intermediate between mesosiderites and pallasites of Group II have already been mentioned. The aubrites and enstatite chondrites of Group I are related directly through the so-called cumberlandites, these consist of fragments of both types, and are the polymictic breccias of Wahl [18]. There are also forms transitional between pallasites and siderites, such as the Breham pallasite.

All of these individual examples, whose number may be multiplied, are cited to demonstrate the promise of this new classification in the understanding of meteorite properties.

Further study of meteorite chemical composition will refine the proposed classification; first with respect to the position of certain achondrites, and then add new data to it. A great aid to classification will be the study of elements disseminated in meteorites, which will bring forth the regular features in meteorites composition [20].

In conclusion, a few words must be said on the terminology of meteorite types; this is closely related to the basic principle of meteorite classification. Acceptance of a single system of classification and of unified terminology evidently will be decided upon by the International Meteorite Commission. In the meantime, this writer suggests that the Brezina-Prior nomenclature be kept unchanged, as it is the most familiar. At the same time, it might be expedient to incorporate the principal results of this paper by adding to the type name (from that system) of a meteorite, its group number from this classification. Examples are "octahedrite, coarse structured, Group II," or "white spheroidal chondrite Group III," etc. This will make it easy to compare the various meteorite types.

CONCLUSIONS

1. Consideration of the general principles of meteorite classification by their material composition shows the necessity of using chemical composition as the primary criterion.

2. In this respect, the most correct of all meteorite classifications is that of Prior; this, however, needs further improvement.

3. The existence of definite regular features in their chemical composition makes it possible to divide the meteorites of all three classes into six subclasses and five groups. This may serve as the basis for classification.

4. Classification of meteorites by their chemical and mineral composition and structure, results from study of their mineralogical and metallographic features.

5. Certain hypotheses on the origin of meteorites may be made on the basis of certain regular features in chemical composition (Prior's "Group Rule") and structure.

The purpose of this article was to set forth the general premises of meteorite classification by their material composition, and, to suggest a general scheme for such a classification.

The author invites criticisms of the shortcomings in this paper; these were unavoidable because of the complexity of the task.

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THE DISTRIBUTION OF TIN-ORE DEPOSITS WITHIN FOLDED ZONES[1]

by M. I. Itsikson

• *translated by William A. Kneller* •

ABSTRACT

The author makes an attempt to establish the general laws governing distribution in time and position of tin-bearing centers and areas in the course of the geological history of the earth; on the basis of methods of regional metallogenic analysis, developed in the U. S. S. R. by a group of scientists under the guidance of G. A. Bilibin. The author has considered the principal geotectonic and structural geological features and factors controlling the regular occurrence during a certain stage of development of a folded zone, of a certain type of tin-bearing intrusions and, hence, of a certain mineralogical-geochemical type of tin deposits.

Considerable tin concentration evidently did not take place during the initial and early stages in development of folded zones. The middle stages of development, corresponding to the period of transformation of a geosyncline into a folded belt, were highly productive with regard to tin. Tin accumulations are associated with syntectonic batholithic intrusions of acid and ultra-acid "potassium" granites. Tin-bearing batholiths and their linear belts are controlled by regional ancient deepseated fractures, confined either to the points of anticlinorium contact (central upheavals) with the adjacent inner synclinoria or to the contact points of downwarpings with consolidated geoanticlinal zones.

The late stages of development of geosynclinal regions corresponding to the period of consolidation of the folded territory also produced considerable tin concentrations.

According to the depth of the controlling intrusions, the showings of tin occurrences belonging to the late stages of development are subdivided into two natural groups: the hypabyssal and the subvolcanic. Tin ore-bearing areas belonging to the late stages of development, are confined to the following two types of geotectonic environments: 1) the mobilized marginal parts of geosynclines in places of their contact with projections of platforms of intermediate massifs (blocks) and consolidated geoanticlinal structures. A number of large tin ore-bearing areas is confined to structures of late superimposed graben-like downwarpings (complicated by fractures) with a powerful development of terrestrial volcanism; 2) the inner portions of geosynclines, where the tin-ore-bearing areas are confined to structural facial zones of inner synclinoria, more frequently in places of contact of the latter with zones of central uplifts.

The distribution of the world tin resources in the course of geological times is as follows: The Precambrian: 3. 3 percent (Nigeria, Rhodesia, Congo, Union of South Africa); The Caledonian folding: 6. 6 percent (Africa and Australia); The Variscian folding: 18. 1 percent (Cornwall, Erzgebirge, the Pyrenees, Australia, Tasmania); The Kimmeridgian folding: 63. 1 percent (South-Eastern Asia).

Simultaneously with the growth of commercial tin concentrations from the Pre-Cambrian to the Mesozoic, complication of the nature of tin mineralization is observed; from tin-bearing pegmatites in the Pre-Cambrian to complex multicomponent deposits in the Mesozoic and the Cenozoic. --Author.

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S. S. Smirnov in a series of works ([24, 26], and others) has turned repeatedly to the question of the laws of distribution of tin-bearing deposits in separate regions of the Soviet Union (Transbaikal, northeastern U. S. S. R. and the Far East). During the last years of his life Smirnov had noted a structural metallogenetic scheme for the huge territory of the Pacific Ocean ore belt [25], emphasizing the time factor of the largest tin-bearing regions of the world in relation to the outer zone of this belt.

In the ten years that have passed since Smirnov's death, substantial progress has been made in the study of tin, based on abundant factual material obtained through newly discovered and formerly known tin-bearing deposits and regions, and, through general principles of regional metallogenic analyses that have been confirmed [5, 6].

[1] Translated from Raspredeleniye olovorudnykh mestorozhdeny v skladchatykh oblastyakh: Sovetskaya Geologiya, no. 1, 1958, p. 86-113.

Questions of geotectonic divisions of the territory of the U. S. S. R. have been far advanced including the geotectonic positions of the most important tin-ore regions. All this permits a preliminary definition of several general laws of the distribution of tin-bearing regions and districts in time and position.

In this work, an attempt has been made to correlate the origin of mineral-geochemical types of tin concentrations with the following: 1) the historical geological stage of development of the folded region; 2) the typical structural-facies or structural-metallogenic zones; 3) the geological structure; and 4) associated intrusions.

The establishment of similar regional-metallogenic laws, even without information concerning the actual facts of tin-bearing deposits, would permit pure analyses based on deduction of the peculiarity of the history of geological development to enter objectively on the appraisal of the prospects of separate regions and areas, and, to forecast new tin-bearing regions.

THE TIN-BEARING DEPOSITS IN SHIELDS (ANCIENT FOLDED STRUCTURES)

As is known, the most characteristic type of tin deposit in the shields is tin-bearing pegmatite. Considerably less typical are the high-temperature quart veins and greisens or disseminated deposits of cassiterite in host granite. Very rarely there are formed tin-bearing, contact-metasomatic veins (Pitkäranta, Finland) or, specifically, pegmatitic funnel-shaped bodies rich in sulfides, localized in carbonate rocks (Arandis, southwest Africa); and, as well, peculiar sulfide-tourmaline high-temperature vein types of deposit located at Mutuye Fides-Stavoren Tin Fields in the Transvaal.

Especially remarkable is the close geochemical (metallogenic) association of tin with tantalum, niobium, and particularly with lithium (the large lithium region Bikita in southern Rhodesia). A less marked association is noticed with rare earths, copper (Katanga), or gold (Western Australia). The rare appearance of tungsten in this association is characteristic. Fragmentary data which afford evidence that tin-bearing regions tend to be at the extremities of shields subjected to faulting (Manitoba, western Australia?), or more commonly are located in sections of the shields activated by recent movements. Thus, in the Belgian Congo, northern Rhodesia, and possibly southwest Africa large tin-bearing areas are localized within the confines or along the extremities of a peculiarly superimposed, somewhat more mobile, structure. The latter extends in a southeastern direction from the Gulf of Guinea to the middle course of the Zambezi river, forming the gigantic "folded apophysis" Caledonite,[2] intruding into the structure of the shield. This mineralization is localized either in displaced rocks of lower Paleozoics and their fractured granites, or, in beddings of Precambrian-basement detrital rocks.

Considerably more clearly noted are the factors of magmagic control. Appearances of tin-bearing deposits are spatially and genetically linked with acid and ultra-acid granitoid intrusions. Expecially characteristic are tourmaline pegmatoid granites, alaskite granites with granophyric and pegmatitic facies, rarely rapakiwi-granites. Especially noteworthy are two peculiarities of tin-bearing granites of this type:

1. The intrusions that appear to be relatively much later manifestations of granitic magma within the shields; if there are even later appearances of granitic magmas, as a rule they are represented with basic or, occasionally, alkaline intrusions of the platform type.

2. Intrusions that are formed at moderate or shallow depths. The presence of granophyre and porphyry facies, association with quartz porphyry and sometimes the spilitic intrusive bodies, give evidence of the shallow depth of tin-bearing intrusions within the shields. In view of the criteria noted, it would be expedient to thoroughly analyze material along the Karelian-Kola border of the eastern-European shield and especially in the Siberian platform. The active extremities of the shields and separate flexures subjected to Caledonian and later movements with the development of explosive disturbances and late acid intrusions, deserve special attention as possibly being tin-bearing.

TIN DEPOSITS IN GEOSYNCLINAL REGIONS

Deposits, related to geosynclinal regions and their marginal zones, yield a basic portion of the world production of tin. Here, more or less, appear all of the known genetic types of concentrations of this metal. However, their distribution in time and space during the process of geological development of the geosynclinal region is exceedingly irregular. Later, on the basis of the analysis of available data on tin-bearing regions of the U. S. S. R. and beyond the frontiers (Burma, Malaya, Indochina, Bolivia, Japan, Australia, and the Varis-

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[2] On regional tectonic diagrams this structure is shown as a region "Of inner-platformed Caledonian folds" (according to A.N. Mazarovich).

cian tin belt of Europe) an attempt has been made to examine the peculiarities of the tin concentrations during development of the geosynclinal region from the beginning and early stages up to the final stages.[3]

Tin-bearing Deposits of the Beginning and Early Stages of Geosynclinal Development

Intensive progressive submergence of the geosynclinal basin is responsible for these stages, accompanied by the formation of specific marine sediments and volcanogenic formations, i. e. spilite-keratophyre, jasper, and siliceous shale, with subordinate limestone and sharply subordinated development of terrigenous sequences. Typical intrusions of ultrabasics hyperbasics and gabbroids are accompanied by large concentrations of chronium, platinum, iron, titanium, nickel, and, to a lesser degree, copper.

With the beginning and early stages of geosynclinal development, the existing tin concentrations actually were not connected. The moderate content of tin or sporadic occurrences of cassiterite grains have been established only in several pyrite deposits (Gumerovskoye, Buron). Probably one of the few examples of existing industrial tin-ore deposits formed in the early stages of geosynclinal development, are the complex tin-bearing, copper-and tin-bearing-polymetallic deposits of Japan, especially those of the Ikuno-Akenobe mining region. These deposits are associated closely with the Neogene subvolcanic intrusions and lava flows of acid and moderately acid composition.

Tin Deposits of the Intermediate Stages of Geosynclinal Development

These stages correspond to the tranformation period of a geosyncline into a folded structure. Most characteristic is the accumulation of large terrigenous (sandy-shale) sequences and carbonates in separate sections as well. The intrusion of large batholiths of moderately acid composition, and later of acid, or, in places, ultra-acid composition is connected with intensive folding.

With the complex of moderately acid granitoids of the early phase of batholithic intrusions, are connected skarn deposits of wolframite and scheelite molybdenite, and quartz-vein, gold-ore deposits commonly associated with arsenic and tungsten (scheelite). With later acid and ultraacid granites the following are genetically connected: pegmatites with minerals of lithium, beryllium, tantalum, niobium, cesium, rare earths, and tin; and, high-temperature quartz-vein deposits of tin, tungsten, molybdenum, bismuth, and arsenic.

The intermediate stages of geosynclinal development are very productive of tin. The predominate part of world tin production is connected with indigenous deposits of this stage, and especially with their distribution. In one form or another, tin occurs with both of the granitoid complexes mentioned here.

With the moderately acid granitoids of the early phase of batholithic intrusives (granodiorites, quartz-diorites, quartz-diorites, adamellites, etc.) are connected small tin-containing sk·rn deposits, i. e. magnetitic sulfide-magnetite and scheelite (tungsten). Extremely fine-grained cassiterite occurs here in close concretion with mineral ores, especially with basic silicates, and usually does not yield easily to extraction. In several cases tin, other than casserite, enters into the composition of garnet in skarns. Ore appearances of a similar type are known in Alaska (Seward peninsula), and, in the Northern Caucasus (Mukulan, Tyutyu-su) as well as on the southern Primore (littoral).

Tin-bearing deposits of the large syntectonic batholiths of acid and ultra-acid "Potassic" granites have immeasurably greater importance. For the sake of brevity, we shall designate this complex as tin-bearing complex "A"; Most typical of this complex are biotite and two-mica, tourmaline, alaskites, and leucocratic and pegmatoid variations of granites. Characteristic is the abundance of dikes and pipes of granite-porphyry, quartz porphyry, and especially aplites and pegmatites, with a relatively uncommon appearance of lamprophyres and dikes of basic-rocks.

In those sections of intrusives contiguous to the roof and as well in separate cupolas and satellites, [of extensive development, there are occasional areas of greisen, (?)]. Activity of type "A" intrusives on the country-rocks is expressed by orogenesis and skarn formation. A stratum of injection or metasomatic granitization of the country-rock is characteristic for deep-seated intrusives.

Tin deposits in connection with these intrusives is fixed essentially to three genetic groups: a) with tin-bearing pegmatites, b) to high-temperature deposits of quartz-cassiterite, and c) metasomatic deposits in carbonate rocks.

Tin-bearing pegmatites that usually belong to the natrolite-lithium type are characterized by intensive superposition of albitization and greisenization processes. A close association of tin with beryllium, lithium, tantalum, niobium, cesium and rubidium is characteristic. In distinction from their analogues formed in

[3]Characteristics of typical stages in development of folded areas (according to Yu. A. Bilibin with additional information by the author) are shown in Table 2.

the shields, content of tungsten (scheelite and wolframite) was noted in the tin-bearing pegmatites examined. Examples of a similar type of ore-appearances are extremely numerous both in the Soviet Union and abroad (Turkestan mountain range, northeastern Kazakhstan, eastern Transbaikal, northeast Asia, the Far East, southeast China, and others).

High-temperature deposits of quartz-cassiterite of the intermediate stages of development are represented by all well known mineral-geochemical types, i. e. greisen, quartz-topaz, quartz, and others. Metasomatic and vein-accumulated mineralization is developed in places, embracing large masses of granite (for example, Blyu-Tyer deposit in Tasmania).

Very close association of tin with tungsten (most often with wolframite) is a most distinctive property of these deposits which are concentrated either in the same deposits or in independent deposits, and, are controlled by the same intrusives. In addition to tungsten, tin is associated in deposits of this type with bismuth, arsenic, and, more rarely, with molybdenum, beryllium and lithium.

Deposits of a different type, connected with the group of deposits examined, yield the predominant part of world production of tin obtained principally in the mining areas of southeast Asia (Malaya, Indonesia, and Thailand). Under the conditions of development of carbonate rocks, metasomatic formations originate either funnel-shaped or complex, irregularly shaped bodies where cassiterite is accompanied by arsonopyrite (löllingite) or scheelite. Similar deposits are very widespread in the tin-bearing provinces of southeast Asia (at Kinta, Malaya, especially the Beatrice, Lakhat, and other mines; at Gedzyu, southern part of the province of Hunan and others), and they are known also in the borders of Central Asia (Maikhura, Takphon, Kobuty, Farkhob) and other regions.

As pointed out, the tin-bearing regions and areas are associated closely with large and very large granitic batholiths which are found to be related to their preroofed [prikrovelnym] parts, and outer contact zones; they are separate roof pendants (cupola) and large blocks of country rock, included in the granite. Also characteristic is the relation of deposits to isolated smaller granite intrusives, or, as linear chains which are disposed along the front of the batholith at some distance from it.

The orthodox point of view holds that intrusives of this type are localized exclusively in the axial regions of central uplifts ("controlled by anticlinoria, batholiths commonly are discovered in the most lifted parts of the latter, in accumulations located at their hinges....." V. V. Belousov [2], p. 453.) However, analysis of materials in actual tin-bearing provinces and regions discloses a more complex picture.

In various geosynclinal systems and regions, locations of the batholiths are very different. The location of tin-bearing batholiths in axial parts of the central upheavals, i. e. in regions of predominant development of overturned structures, is not so commonly observed. More usual is the control of intrusive bodies by downwarpings, which have undergone submergence in the intermediate stages of development. Usually, this downwarping is compensated by an accumulation of very thick terrigenous (sandy-shale), more frequently of flyschoid sequences, and, in small measure, of limestone. Such a type of tectonic development corresponds to zones of intrageosynclinoria,[4] during the period of the outset of their fracture by attenuated central upheavals on the border of related, resultant downwarpings.

Tin-bearing batholiths are localized for the most part where anticlinoria are hinged to contiguous inner synclinoria, including also the the adjacent limbs of these structures. Also typical is their location along fractures in the geanticlinal zone, that had undergone little movement and no submergence of the lower structural stage and pregeosynclinal foundation. The accumulation of more and more data indicates that, in addition to numerous structural anomalies, the chief factor in the emplacement of tin-bearing batholiths and their linear belts is regional, long-extant deep fractures or structural lineaments accompanied by folding. Later, this question will be considered in more detail in a special section. Here, we shall bring forward only a few examples.

The huge Indonesian-Malayan-Burman tin-bearing (tungsten-tin) belt may serve as the most clear example of control of a tin-bearing region by a large structural seam. This belt extends more than 2500 km from the islands Billiton and Banka to the Tavoy region on the Malacca peninsula; and evidently continues further north almost to Rangoon. Along this trend, tin deposits are connected with a belt of granitic batholiths, whose minerals correspond to Kimmeridgian folded movements.

The following factors attest convincingly to the controlling importance of large structural lineaments (deep fractures) in localization of intrusives and tin deposits: 1) the control of tin-bearing batholiths of this belt by "jointed" zones; 2) the disposition of batho-

[4] Here also, lower (simple) tectonic terminology is needed in order to understand V.V. Belousov.

liths as a great, attenuated, and nearly uninterrupted ribbon extending from one structural facies zone to another; and 3) the conjunction of this ribbon with linear strips of recent (upper Mesozoic-Paleocene) dolerites and quartz-porphyries.

Also indicative of this association is the structural position of Kalba-Narym batholith, near Irtysh. This large intrusive, with its tin-tungsten production is situated in the sandy-clayey sequence (Takyrian Suite) D_3 - C_1, principally along its contact with the underlying suite of Kystav-Kurchumskian suite D_2 - D_3 [13]. The following Dalam-Karynskian zone of tuffaceous sandstones C_1^2 further to the southwest, is characterized by gold mineralization. Along all of its more than 300-km extent, the batholith, near Irtysh, is accompanied strictly by great zones of fracture located to the northeast. Here, also the intrusive is not related to the axial crest of the anticlinorium. It is located at its margin, close to the junction with an adjacent downwarping, and, is sharply controlled by a regional, prolonged, active fracture.

Still more characteristic is the structural position of batholiths (supposedly upper Jurassic) of the Kolymo-Indigirka region; they form a gigantic chain that borders the periphery of the Central Kolymian massif. Most batholiths are localized predominantly in the Jurassic, and partially in the Triassic, intrageosynclinal downwarpings; and, they are concentrated in a series of regional fractures bordering the periphery of the Middle Kolymian massif. Similar examples may be enumerated. They all agree in attesting that tin-bearing batholiths of the intermediate stages of geosynclinal development are associated with the following: 1) inner parts of the geosyncline; 2) structural facies zones of central upheavals, i. e. anticlinoria, inner synclinoria; 3) fractured parts of these two structural-facies zones; or 4) fractured parts of downwarpings within the intrageanticline. Emplacement of the intrusives is controlled protractedly by extant, deep regional fractures that developed simultaneously with the folded structures and continue its development later, in a period of transformation of the geosyncline into an accretionary rock structure.

Tin-bearing Deposits of the Late and Final Stages of Geosynclinal Development.

These stages correspond to conditions of progressive attenuation of the tectonic movements, finally resulting in consolidation of the folded region and its transition to a new platform. The tectonic regime is characterized by a waning of folding movements, and, by sharp growth of intense, explosive fractures that break through the consolidated structure. In a number of cases, grabenlike fractures bordered by large structural seams (deep fractures), were formed. Associated with these fractures is the appearance of extensive terrestrial volcanism and accumulation of volcanogenic and terrestrial-freshwater sequences, which in some places are coal-bearing.

In addition to tin, gold and molybdenum are of leading importance in the metallogenesis of the late stages of geosynclinal development, while copper, lead, zinc, and iron (magnetite) are of lesser importance.

It is essential to emphasize that actual concentrations of tin are not connected with the final stages of development. It is possible that these geotectonic environments are partially responsible for small deposits of colloidal cassiterite, i. e. wood-tin, occurring with acid effusives (liparites, rhyolites) in a series of recent folded areas (Mexico, Nevada, New Mexico, and others).

These stages in development are characterized by a variegated suite of mineralization including antimony, mercury, lead-zinc-copper (telethermal deposits), fluorite, barite, hematite, and siderite; and , in various structural-metallogenic zones, silver-cobalt-nickel-arsenic, and others.

Later stages in geosynclinal development are productive in tin and include approximately 20 percent of world tin resources. In size, they are second only to deposits of the intermediate stages. Tin deposits of the Soviet Union are connected with the later stages in geosynclinal development.

The various types of tin-bearing deposits are formed in geotectonic environments of the late stages and may be subdivided into two groups formed under several different geological conditions and controlled by different types of intrusives: 1) the hypabyssal type and 2) the subvolcanic (near-surface) type.

First Group - Hypabyssal

The group is characterized by a genetic connection with intrusives of tin-bearing granitoids such as the so-called type "B". These hypabyssal and subvolcanic spilitic intrusives are considerably rarer. The most characteristic factor is that they were intruded in an already prepared, folded belt forming a more or less consolidated structure. They occupy cataclastic zones, zones of tectonic weakness, or planes of exfoliation between rocks of an upper structural level and the attenuated, intensively dislocated formations of a pregeosynclinal basement.

The intrusives that we have examined range in composition from diorites or monzonites to granodiorites, granites, and, in places, alas-

kites. As a rule, sequence of the formation of separate members in the comples ranges from more-basic to acid varieties.

In one of the tin-bearing regions[5] a sequence of rocks (given from much older to younger) has been established: 1) quartz-diorite; 2) monzonites-granodiorites; 3) granodiorites and hornblende granites; and 4) granites and alaskites (negligible). With regard to tin deposits, there are examples of 3) and 4) only. In the other part of the same region, a somewhat different sequence is evident: 1) diorites, diorite porphyry, and quartz-diorites; 2) monzonites; 3) quartz-monzonites; 4) monzonite-granodiorites; and 5) tin-bearing granites.

For the series of regions, a predominant connection was observed of tin-bearing deposits with moderately-acid intrusives of the "anomalous andesine granite" or "hybrid granite" type, with a general trend towards the composition of tonalite (Bureinian mountain chain and others).

In regions of northeastern Yakutia [16] occurrence of metals is connected with diorite, quartz-diorite, and monzonite intrusions. These rocks are regarded as products of assimilation and hybridism. In composition the rocks are close to prophyritic granodiorites and, to a lesser degree, to quartz-diorites and andesine granites, alaskite and leucocratic varieties of granites characteristic of the Pevek Peninsula. In a neighboring region, tin-bearing granites belong to the normal type [9]. Tin-bearing granites of type "B" have characteristics that are sharply distinguishable from those tin-bearing intrusives of the intermediate stages (tin-bearing batholithic intrusives of type "A"), where, in fact, there is a complete absence of pegmatites. Even with the exceptional richness of ultra-acid magma in volatile constituents and in water its pegmatite-forming potential is expressed originally as tin-bearing miarolitic granites, where cassiterite, topaz, and feldspar are formed in cavities (miarolites), in the granite (southeast Asia, Southern Sikhote-Alin). Aplites here also occupy a limited area. In addition to these, extremely varied dikes and apophyses of granite-porphyry and quartz-porphyry appear with relatively limited development of dikes of basic rocks and lamprophyres.

The influence on country rocks of tin-bearing intrusives of the type "B" usually is quite intensive and unusual. Besides orogenesis, especially typical is the wide manifestation of processes imposed on the country-rock by pneumo-hydrothermal processes; and especially in roof areas, by large zones of tourmalization, sulphidization, silicification, or sometimes greisenization. In a series of instances, characteristic zones of biotite-siderophyllite rocks with varying amphibole content of the hastingsite or actinolite groups are found. Similar areas of iron-potassium or sometimes iron-boron metasomatism commonly are projected on the surface as large funnel-shaped bodies, directed upwards, over still unrevealed eroded intrusives. In several regions, this peculiarity is used successfully to search for indications of tin-bearing areas.

Two mineral-geochemical branches of tin-bearing mineralization are located in genetic and spatial association with intrusives of type "B": 1) Deposits of sulfide-cassiterite, predominately iron-bearing series (according to classification of E. A. Radkevich) represented by their various sub-types; principally tourmaline-sulfide, chlorite-sulfide, and similar deposits which in their extremities sometimes change into lead-zinc-tin-containing deposits. 2) Deposits of quartz-cassiterite and quartz-wolframite with subordinate content of molybdenite, bismuth and arsenic, i. e. veined, rod-like, greisenlike zones, etc., are exceedingly similar to the mineral-geochemical properties already characterized by wolframite-tin-bearing and tin-ore deposits described above, connected with batholiths of type "A" of the intermediate stages of geosynclinal development.

Deposits of each of these branches are found in a series of regions closely joined together by transitional ore-manifestations. In some cases, a clear zonal disposition of ore, i. e. manifestations of a different mineralogical-geochemical type, can be determined with respect to intrusives. Thus, in one of the regions of Chukotia [1] there are immediately located, in the granites, quart-topaz veins, and greisens; in the direct contact zone, there are mineral bodies of quartz and tourmaline; and, in an outer contact zone, there are quartz veins with iron-bearing chlorite, magnetite, and hematite.

Clear examples of zonal distribution may be seen in one of the regions of Central Sikhote-Alin. Here, related to small, leucocratic-granite intrusives, only slightly eroded, and to their immediate contact zones, are greisen and vein zones of wolframite, cassiterite, and scheelite; and, in places, molybdenite, bismuth, and arsenopyrite. Cassiterite-tourmaline ore deposits are located in the outer contact zone. In the borders a few kilometers from the contact are located veins and splintery zones of tourmaline-chloritic and quartz-chloritic sulfide mineralization. Lead-zinc ore deposits not containing tin, are established at the zone of peripheral ore-bearing concentrations.

[5] According to investigations of E.P. Izokh.

In spite of the apparent similarity of late stage quartz-cassiterite deposits to the intermediate stage type deposits associated with the batholith intrusive of type "A", there are elements that differentiate them. The most important elements are: a) Geological conditions that control tin-bearing intrusives are different. b) Late stage quartz-cassiterite mineralization is completely foreign to pegmatites. This mineralization is interwoven closely in space and time with sulfide-cassiterite mineralization, with which the transitional links are connected; the intermediate-stage, quartz-cassiterite mineralization is associated quite rarely with a sulfide-cassiterite suite, and, is related very closely to tin-bearing pegmatites. c) Several differences are noticeable also in the geochemistry of the quartz-cassiterite ore deposits of comparable groups.

Although it may not be true everywhere (this needs special analysis) in late stage deposits, the role of lithium and beryl is lessened somewhat more than is expressed intensively in quartz-cassiterite suites, and, even more so, in pegmatites of intermediate-stage deposits.

Second Group - close to the surface (subvolcanic)

This group of late-stage tin-bearing deposits is formed even shallower conditions than is the first group. It is clearly associated with small, near-surface subvolcanic or fissured intrusives of the so-called type "C", that in most cases gravitate extensively to zones of effusives.

According to their form, intrusives of type "C" most commonly are represented by dikes; dike suites, coulisse-shaped in arrangement, or branched pipes; inter-layered intrusions, transforming at depth into vein-like or funnel-like bodies; and, mushroomlike or axelike, narrowing with depth as in ethmoliths and neck type intrusives. A most characteristic structure of several intrusives in this group is the presence of an unusual brecciated texture typified by a "mixture" of fragments of the intrusive with fragments of the country rock (Llallagua, Oruro, and other "volcanized" tin-ore deposits of southern Bolivia, Little Hsingan, western Sikhote-Alin, and others). Similar mixed breccias, usually associated with mineralization, may originate as explosion breccias resulting from sudden gaseous break-through along small fissures connecting these subvolcanic formations with the surface.

Tin-bearing intrusives of type "C" embrace a wide range of rocks that are of acid, often moderately acid, or sometimes of intermediate composition. The rock types range from felsite-porphyries, quartz-porphyries (granite-porphyries), and granodiorite-porphyries, to quartz-diorite porphyries. The contactual influence of intrusives on country rocks is exceptionally weak. In addition, the following are particularly characteristic: wide areas of hydrothermal change commonly disportionate with the size of the intrusives, both in the in-intrusives themselves (autometasomatic type), as in the country rocks (chloritization, sercitization, silicification, and sulfurization).

The relationship between separate varieties of intrusives and that between intrusives and mineralization, is quite complicated and changeable from region to region.

The facts brought forward obscure the close spatial association of tin deposits with a complicated series of small intrusives of type "C", and determine the position of mineralization as some intermediate link in the complicated and repeated, but generally single, intermittent process of the formation of a dike complex and mineralization. In addition, the fact of spatial autonomy of mineral bodies, and their isolation from intrusives in deeper sections, provides a basis to interpret the association of mineralization with intrusives as actually paragenetic. In this consists the profound difference of tin-bearing intrusives of type "C" from other, previously investigated, productive, tin-bearing intrusives of complexes "A" and "B" in which there is no doubt about the genetic connection of mineralization.

The second principal difference is that the ore-formation process associated with intrusives of type "A" and "B" originates in a closed physicochemical system. Ores associated with intrusives of type "C" originate in a half-open physicochemical system controlled by the relationship of the ore-formation chamber to the subareal sphere [lower atmosphere], a condition that permits a sharp loss of pressure and short-lived ore-deposition processes, etc. This ore genesis leaves an imprint on ores formed close to the surface, giving them features of unrepeatable originality. Here is formed a varied group of tin-ore, silver tin-ore, and tin-polymetallic deposits of different types of sulfide-cassiterite deposits belonging both to an iron-bearing and a lead-zinc series [20]. Most characteristic are representatives of the chlorite-sulfide type of deposits with different proportions of iron-chlorite and sulfides, and the so-called near-surface deposits with sulfides of lead, zinc, and sulfostannites, often with an important admixture of silver. Characteristic features of the deposits are telescoping deposition of different mineral-geochemical facies one upon the other, presence of mineral associations with metastable components, and indications of ore-deposition from colloids.

Silver-lead-zinc ores are found commonly in metallogenetic association with tin-bearing deposits and with their associated transitional

members connected with them. Less often, ore deposits of antimony and bismuth are located in replacement by removal of one or the other. Tungsten, although not alien to tin-ore occurrences of the subvolcanic group, is found rarely in industrial concentration.

It still remains to examine the question of the geostructural location of tin ore belts, regions, and deposits formed in the late stages of the development of folded areas.

Two principal groups of tin-bearing geostructural positions are distinguished according to their location in a geosynclinal region: 1) in the marginal parts of the geosyncline, and 2) in the inner parts of the geosyncline.

To the first group belong the tin-ore belts and regions disposed at the junction of mobile marginal parts of the geosyncline (usually along large fractures) with protruding platforms, central massifs, or consolidated geanticlinal structures.

Where associated with geosynclines having advanced (sometimes foothill) downwarpings in molasse, lagoonal, or caustobiolithic sediment accumulation, tin-bearing zones are situated more or less parallel to them, usually on the side nearest the geosyncline. Here, as a rule, the foundation or consolidated basement stage is exposed because it was subjected to the influence of large regional fractures.

Also exceedingly characteristic is the close relationship of the tin-bearing belts and regions to the fringe of consolidated structures of the central-massif, block, or inner-platform type. Tin-bearing regions of northeast Asia, markedly related to the fringe of the Central-Kolymian middle massif blocks [26], may serve as an example of such disposition.

Finally, one of the characteristic structures of the late marginal type, favorable for the localization of tin-bearing regions, consists of late graben-shaped downwarpings with intensive development of surface volcanism. These peculiar "volcanic" dwonwarpings (volcanic belts) complicated and bordered by large structural lineaments (with deep fractures), are characterized by a large accumulation of volcanic products and sometimes by surface freshwater, and carbonaceous strata. These downwarpings may be examined as analogous in type to advanced downwarpings of provinces where the former are absent or not sufficiently developed; for example, in several geosynclinal systems of east Asia.

Analysis of the structural position of corresponding tin-bearing regions shows that these volcanic downwarpings are located most frequently in terminal parts of geosynclinal zones at the place of their transition to nonmobile regions of platforms or to earlier consolidated geanticlines. Sometimes downwarpings and, evidently, fractures controlling them, are located at an angle to the attenuated under-folded structure, cutting through it obliquely. The late Mesozoic Singano-Okhotsk volcanic belt (downwarping) near the Amur, is situated at the junction of the Turonian (Zee of Borani) of the central massif on the western side of the east Asiatic (Sikhote-Alinian) geosyncline (research of M. I. Itsikson).

A Cretaceous-Paleogene coastal belt of the Southern Maritime Province occupies also a similar position along the eastern border of the same geosyncline at its junction with the Chinese Platform, now submerged under the Sea of Japan.

Cutting at an angle, the attenuated lower-Middle Mesozoic folded structure protrudes more than 300 km into the folded region of the Omsukchanian Upper-Cretaceous volcanic downwarpings along borders of the Soviet northeast (research by V. T. Matveyenko), [the structural lineament of western Sikhote-Alin (Belyayevsky, N. A. [3])].

Tin deposits peculiar to geostructural positions of the marginal type, represent both of the previously discussed groups of hypabyssal and terrestrial (subvolcanic) occurrences.

Geostructural positions of the inner type, correspond to structural-facies zones of large inner synclinoria that have great significance in the localization of several types of tin-ore belts originating in late stages of geosynclinal formation. The tin-ore occurrences of intermediate stages are localized in similar structures, and, are connected with batholithic intrusives of type "A".

However, in the late stages of geosynclinal development the circumstances change; and small tin-bearing intrusives of type "B" were intruded after the completion of the main folding of the consolidated structure. Typically, the location of these intrusives and corresponding nodal or chain-like ore deposits, frequently are orientated obliquely to folded structures (Verkhoyansk, central Sikhote-Alin). At the same time, the entire disposition of extended ore belts, represent an aggregate of similar chains joined concordantly to a corresponding folded structure.

Detailed study of ore belts discloses a not yet completely known complex of regional structural lineaments that control ore deposition. These lineaments are located along the junction of heterogeneous structural-facies zones that have experienced sharply contrasted

fluctuating movements. The movements commonly occur between the central upheaval and adjacent downwarpings. Chains of intrusives and their associated ore bodies are localized predominantly along the margins of the downwarping. Emplacement of the intrusives are controlled, perhaps by extended "blind" fractures in the attenuated pre-geosynclinal foundation or lower structural level.

For the inner geostructural positions in late stages of geosynclinal formation, the most characteristic are tin-ore deposits of the ferruginous series and the whole range of transitional members up to the quartz-cassiterite, quartz-wolframite, and, frequently, greisen deposits. This complex of mineralization is associated with the hypabyssal tin-bearing intrusives of type "B". It is essential to emphasize that terrestrial tin-bearing deposits of the subvolcanic intrusives of the type "C" are by no means uniform in their occurrence. The only uniformity is that these deposits are formed in marginal geostructural positions.

Several tin-ore regions (southern Bolivia, western Verkhoyansk) formed in the late stages of geosynclinal development, aside from the geostructural position dependent upon them, reveal the control on places of bending or sharp turns by the regionally folded structure. Thus, in the Bolivian belt the northwest-trending structures (Cochabamba-Colquechaqa region bent to the south and, in places, to the northeast (from Kolkvechak to 22° s. latitude) forming a protuberance into the side of the Brazilian shield; probably, the structures are associated with curvilinear fractures bordering the outline of the shield. As E. A. Radkevich justly points out, curves of the folded structure, in the presence of other signs, are favorable structural areas for the localization of tin-ore regions.

THE ROLE OF DEEP FRACTURES IN THE LOCALIZATION OF TIN-BEARING REGIONS

On the basis of the latest data of geophysics, seismology, and detailed structural-facies analysis the question was raised in recent years concerning the great importance of deep fractures on various geological processes (sediment accumulation, tectonics, and magmatism). This was especially emphasized in the works of A. V. Peive [17, 18], N. S. Shatsky [28], A. N. Zavaritsky [9], V. A. Obruchev [14], and others. From the foregoing account it appears difficult to assess the controlling role of deep fractures in the localization of tin-ore belts and regions.

Having summed up the material stated in the foregoing pages, we shall examine the form and place of the manifestation of deep fractures in connection with tin-bearing deposits, at different stages of geosynclinal development.

Not considering the beginning, early stages of geosynclinal development that are low in tin productivity, we shall pass immediately to the intermediate stages. In this case, fractures are localized predominantly in the inner parts of the geosyncline; and, are controlled by the linear granites of heterogeneous structural-facies zones. According to existing classification, mineral belts controlled by fractures of this type must belong to the accordant types [23]. These fractures were unexposed to the earth's surface at the time they were formed. Their activity affected mainly the foundation and lower structural levels of the folded structures. However, as a result of these deep structures, the fractures exhibited a more intensive influence on the upper structural levels, influencing the degree of the submergence of flexures and the lithofacies composition of sediments.

A property characteristic of the development of deep fractures in the intermediate stages is a most intensive manifestation of deep magmatism in the form of granitic batholiths. In fact, extended ribbons of granitic batholiths lying on zones of a very large crust of permeable earth and along a seam of fracture, project very clearly the direction of fracture on the surface of a contemporary eroded section. Tin-bearing deposits most often associated with acid and ultra-acid varieties of the granite type "A", are represented by cassiterite deposits, pegmatite and quartz-cassiterite formations and associated deposits. The large Indonesian-Malayan-Burman belt is the clearest example of tin-bearing provinces with this type of control.

In the process of the subsequent development of a geosyncline, i. e. in its late stages, the greatest tectonic activity is controlled by zones of marginal parts of the geosyncline; where late flexures most frequently all migrate. Along the junction between the geosynclinal margin and the platform, large systems of deep fractures are developed at this time; and, control these flexures and determine the intensive manifestation of terrestrial volcanism.

In several geosynclines, however, these late downwarpings and their concentrated fractures are developed at the terminations of central massifs or are laid up on earlier consolidated parts of the upheavals. At the junction of the folded belt with the central massifs, the late deep fractures also are clearly manifested.

Such a type of fractures is profoundly different from fractures formed in the process of preceding stages. Late fractures, in contrast from "blind" fractures of the intermediate stages, reach high structural levels and influence them most intensively. These fractures are noticed with powerful and extended volcanized belts; closely associated with them

are small, tin-bearing terrestrial and hypabyssal intrusives of types "C" and "B".

Several tin-bearing volcanic belts in the Far East, convincingly prove their connections with regional structural seams [3, 4, 10]. Another proof of the regional character of the fractures is their extent over many hundreds of kilometers and their structural position at the junction of various geotectonic elements over extended periods of geologic time. The latter is confirmed by different ages of volcanic activity (upper Permian, Upper Jurassic, Lower Cretaceous, and Upper Cretaceous) that extend along these same fracture zones.

In several Far eastern tin-bearing regions, the author directly observed sections of fractures in their natural state. These sections are stretched along the regional fracture with intensive development of upper Mesozoic volcanites and represent zones up to 10 km wide. They are exclusive of strong brecciation of ancient complexes, and, are hidden by effusive coverings penetrated by a branched-out net of dike rocks.

Besides the formation and activization of late, marginal deep fractures, there occurred also an enlivening along old seams of the inner parts of the geosyncline in the late stages of geosynclinal development. The greatest activity occurred in the intermediate stages of development. In the period investigated, however, this activity was predominant in a complicated form of motion along the fractures which cut obliquely through the folded structure of the foundation. These fractures are projected as chains of tin-bearing intrusives of type "B" which intersect the consolidated folded structure at an oblique angle.

As was already pointed out, with intrusives of this type are associated tin-ore deposits of the iron-bearing series of sulfide-cassiterite formation, quartz-cassiterite and various transitional types. M. P. Rusakov [21], G. P. Volarovich [7], M. P. Materikov [12], and other investigators have emphasized the great significance of fractures in the development of local structures of tin-ore bodies and separate deposits of the coastal region.

Abyssal fractures (structural seams) controlling tin-bearing deposits, may be mapped and studied either by their direct manifestations that resemble large tectonic [zones] of regional size or, more frequently, by their marked volcanic belts, garlands of tin-bearing intrusives, or weakened zones (zones of the highest permeability) with intensive dike-complex development. Fractures of intermediate stages are noticed as attenuated ribbons of regional scale, or chains of granite batholiths with their concomitant and closely associated tin and tungsten tin belts.

These fractures regularly arise at a definite stage of the development of a folded structure and occupy in it a completely definite position, both in time and place of occurrence. In essence, the structurally controlled ore bodies under investigation have little in common with the completely hypothetical gigantic fractures of the ore-canal type ("The Great Silver Canal of America" (The Ore Magmas), J. E. Spurr [35],), which scoop up metals from the unknown depths of the submagmatic zone. Spurr's hypothesis is just as unfounded as the geologically based fractures "with air-holes" of Locke, Billingslay, and Schmitt [32].

The data stated above concerning those properties of abyssal fractures that control tin-bear-

TABLE 1. Characteristics of Abyssal Fractures controlling the distribution of tin-bearing belts and regions

Abyssal Fractures of Intermediate Stages	Abyssal Fractures of Late Stages
Geostructural Position	
Developed predominantly in inner zones of the geosyncline.	Developed more frequently in marginal parts of the geosyncline, at the edge of the central massifs, and in late superimposed "volcanic" downwarpings.
More frequently discordant with the folded structure.	Discordant and cross-cutting in relation to the folded structure.
Influence of Fractures with Depth	
"Blind" fractures reach the foundation; on the surface and in the upper structural stages, they are not manifested directly.	"Gaping" fractures reach the surface and immediately act most intensively on the upper structural level.
Magmatism	
Granite batholiths (Tin-bearing intrusives of type "A").	Terrestrial lava effusions of acid and intermediate composition. Tin-bearing hypabyssal intrusives of type "B" and subvolcanics of type "C".
The Character of the Fixation of Fractures on the Surface	
The attenuated belts (ribbons) of granite batholiths, mainly along linear borders of varied structural-facial zones.	Tectonite zones of regional scale and accompanying volcanic belts. Garlands (chains) of small tin-bearing intrusives. Zones of weakening (with increased permeability) with a development of a dike complex.

ing belts and regions, are set out on Table 1.

CLASSIFICATION OF THE GEOLOGICAL CIRCUMSTANCES DETERMINING THE FORMATION OF TIN-ORE DEPOSITS

Data on the time and place of tin-deposit formation in the developmental process of folded regions, stated in the preceding pages appears in Table 2. This table represents an attempt to systematize the chief geological-structural properties which determine the mineral-geochemical type of tin concentration during stages of geosynclinal formation. The table shows the genesis of ore deposits as it is associated with geological development of a folded region. The table also outlines the localization of tin-bearing regions in the structure as well as their connection with magmatism and with the type of tectonic development, etc.

From Table 2 it follows that the appearance of quartz-cassiterite formations, according to the conditions of its formation, corresponds to two actually different historical-geological and structural-facies occurrences. These deposits form in the middle stages of geosynclinal development and, are associated with "potassium" granites batholiths of type "A". In late stages, these deposits are associated with spilitic late-folded intrusives of type "B". Equally, sulfide-cassiterite deposits generally correspond to late stages, but at times may form in the intermediate stage of geosynclinal development.

Thus, several similar mineral-geochemical deposits are formed in actually different geological environments at different stages of geosynclinal development and, are localized in different structural-facies zones. Consequently, there is actually a difference in the appearances of these deposits and of principles for their evaluation. In most cases, too, exploration techniques used in their discovery are different. Metallogenic division (into districts) for the prognosis and evaluation of perspectives for tin-bearing deposits in regions that have undergone little investigation, (where in a number of cases, actual mineralization generally has not yet been established) classification of typical geological conditions that determine the rise of genetic types of deposits.

Table 2 should by no means be regarded as any kind of new classification of tin-ore deposits. The classification already worked out by Smirnov and developed further by O. D. Levitsky and E. A. Radkevich completely answers this purpose. This classification, exhibiting mainly the mineral-geochemical properties of specific deposits, has very great significance in facilitating ore-deposit exploration and investigation, and, is helpful in the solution of ore genesis problems. Table 2 systematizes typical geological characteristics and anticipates the appearance of certain mineral-geochemical types of deposits. This combines harmoniously with this classification, indicating new evidence that is expecially important on separate, very early stages of the solution of tin deposits [problems] in separate structures and regions.

Table 2 is intended to be of use mainly for the following: 1) regional metallogenic structures, 2) compilation of prognostic maps, 3) interpretation of data of drilling and metallometric testing, 4) evaluation of perspectives of little investigated territories, etc.; i. e. in those regions where no known deposits occur but have favorable geological premises. In these circumstances the "decisive ability" of mineralogical-geochemical classification, based on the knowledge of the actual ore substance, is in fact insufficient; and, factors of geotectonic and geological-structural control acquire the very greatest importance.

DO TIN-BEARING EPOCHS AND TIN-BEARING PROVINCES EXIST?

In 1948 and 1949 geologists attempted to calculate the distribution of known supplies of important metals throughout geological time, as a collective effort of the all Union Scientific Research Geological Institute on questions of metallurgy. The section of research on tin was completed by the author; the results of investigation have been published [7 (?)].

Subsequently, the calculations were done more precisely, and are given below. According to these calculations, the general world resources of industrial ore (except the U.S.S.R.) were found to approximate 12 million T. This included somewhat more than 4. 5 million T of tin ore extracted beyond the historical period reviewed (mainly southeast Asia, Cornwall, the Erzgebirge, the Pyrenees, and Australia). The distribution of world resources of tin in the geological-time section is sketched in the following figure 1 (except the U.S.S.R.): Precambrian folding, 3. 3 percent (Nigeria, Rhodesia, Congo, and Union of South Africa); Caledonian folding, 6. 6 percent (Africa and Australia); Variscian folding, 18. 1 percent (Cornwall, Erzgebirge, the Pyrenees, Australia, and Tasmania); Kimmeridgian folding, 63. 1 percent (southeast Asia); and Alpines (Alpids, 8. 2 percent (Bolivia, etc.).

Thus, considering the erosional destruction of a series of tin-bearing regions of Precambrian and early Paleozoic mineralization; there is, nevertheless, a clearly expressed tendency towards the progressive increase for the role of tin in accordance with geological development of the earth's crust, attaining sharp cul-

TABLE 2. Outline of the tin-ore deposits formed

Stage of development		The general characteristics of the stages (according to Yu. A. Bilibin, with additions)	Type of tectonic movements	The geologic position (emplacement) of tin-bearing regions
GEOSYNCLINAL REGIONS	FINAL	1. Formation of a recent platform. 2. Accumulation of terrestrial-freshwater sediments. 3. Erosion. 4. Crust of weathering (weathering of the Crust?). 5. Terrestrial effusions of basalts, alkaline and acid lavas. 6. Telethermal deposits of lead, zinc, and copper; barite, vitrite, fluorite, iron (siderite).	1. Vaulted upheavals. 2. Graben-shaped downwarpings. 3. Attenuated fractures.	(Tin-bearing deposits are
	LATE	Progressive consolidation of the folded belt: 1. Accumulation of continental and freshwater-terrestrial sediments. 2. In the foreland flexures, the development of lagoons and the formation of molasse deposits. 3. For other geosynclines (east and northeast Asia) there are more typical terrestrial effusions of lava of acid and intermediate composition. 4. At the end of this stage occur effusions of andesite and basalts. 5. Occurrence of shallow spilitic intrusives of moderately acid, acid and rarely intermediate composition. 6. Unique endogenous mineralzations. 7. Leading metals-gold, copper, molybdenum, tin; less typical lead, zinc, and iron (magnetite). 8. Antimony and mercury are characteristic for separate structural and metallogenic zones.	1. Steady uplifting. 2. Progressive attenuation of folded movements. 3. Intensive explosive dislocations. 4. Great activity along deep fractures of the upper structural level. 5. Formation of late superimposed graben-shaped downwarpings along attenuated fractures and with intensive volcanism. In some areas occur deposits of terrestrial sediments with coal measures.	1. Positions of the terminal type: a. They are emplaced in the mobile terminal parts of the geosyncline at their junction with zones of abyssal fractures which are associated with protruding platforms, central massifs or consolidated geoanticlinal structures. b. Late graben-shaped superimposed downwarping with intensive development of terrestrial volcanism, ("volcanic") downwarpings or volcanized belts. c. The downwarpings are often emplaced obliquely in relation to folded structure. 2. Positions of the inner type: a. They correspond to the inner downwarpings of the middle stages of geosynclinal development (in large structures of the inner synclinoria). b. Tin-bearing intrusives of type "B" are emplaced deep in the late stages of development of the already consolidated folded structure. c. Tin-bearing intrusives and ore bodies are located in chains which are aligned obliquely to the folded structure and follow fractures of the foundation (of the lower structural level).

during the geological development of folded regions.

Tin-bearing intrusives	Groups (branches) of tin mineralizations mineralogical and geochemical types of deposits	Metallogenic associations	Economic and refinery importance
manifested; very rare ore appearance, and small deposits of wood tin-rhyolite formations)			
Tin-bearing intrusives of type "C": Small terrestrial (sub-volcanic) fractured intrusives diffusely, and evidently genetically connected with effusives. Forms of intrusives: Dikes, dike swarms, pipes, interlayered intrusives changing with depth into funnel-shaped and layered bodies, necks-ethmoliths Composition: Granite porphyry, quartz porphyry, granodiorite porphyry, and diorite porphyry. Influence on the country-rocks: Contact influence weak; very intensive appearances of metasomatic alterations--Chloritization, sericitization, silicification, sulfidization, sometimes alunitization and kaolinization. Tin-bearing deposits are associated with the intrusives.	Terrestrial group: Includes deposits of different types of sulfide-cassiterite formations, ferruginous and lead-zinc series, tin ore, and tin polymetallic deposits. Especially typical are representatives of the chlorite-sulfide subtype, and sulfide, essentially composed of an admixture of sulfur and silver.	Lead zinc, silver, more rarely boron (datolite, axinite) and arsenic.	Large
Tin-bearing intrusives of type "B": Fractured hypabyssal intrusives. Forms of intrusives: Massifs, pipes, intrusions interlain between rocks of the upper structural level and the attenuated, intensively fractured basement. Composition: Complicated (repeated) intrusives ranging from quartz-diorites, monzonites, and granodiorites to granites. Tin-bearing deposits are connected with the acid members of the intrusive series. Clear indications of hybridism with the formation of "anomolous andesine granites" and "hybrid granites" which trend in composition towards the tonalite petrochemical branch. There is an abundance of accompanying granite porphyries and quartz porphyries (dikes and pipes). Aplites are uncommon and pegmatites are absent. Influence on the country-rocks: Orogenic and intensive pneumatohydrothermal activity form large fields or tourmalinization, sulfidization. The occurrence of autometasomatic greisenization. The formation of new biotite-siderite phillitic rocks with ferriferous amphiboles.	Hypabyssal group: 1). Deposits of a ferruginous series of sulfide-cassiterite mineralization (tourmaline-sulfide, chlorite, chlorite-sulfide and small amounts of sulfide with ferruginous silicates).	Lead, zinc.	1). Very large, most important in the U.S.S.R.
	2). In the endocontact zone of intrusives of type "B" metasomatic greisens and veins of quartz-cassiterite and quartz-wolframite-cassiterite are formed. In the volatiles derived from the intrusives transitional type deposits (tourmaline-cassiterite, topaz-cassiterite-sulfide, etc.) and sulfide-cassiterite deposits are found.	Tungsten, arsenic, bismuth, and rarely molybdenum	2). Insufficiently studied.

TABLE 2. Outline of the tin-ore deposits formed during

Stage of development		The general characteristics of the stages (according to Yu. A. Bilibin, with additions)	Type of tectonic movements	The geologic position (emplacement) of tin-bearing regions
GEOSYNCLINAL REGIONS	MIDDLE	Gradual drainage of the geosyncline and its development into a folded region: The accumulation of large homogeneous sandy-shaley thicknesses ("terrigenous flysch") and more rarely of carbonates. Batholithic intrusions of moderately acid composition (later-of acid and ultra-acid composition) occur along zones of abyssal fractures. With the earlier (moderately acid) phase of the batholith are associated skarn deposits of scheelite and molybdenite. Also associated is veined gold ore.	Submergence at the beginning of this stage--then predominant uplift 1. Most intensive movements. 2. Abyssal fractures in the basement control the disposition of the batholith.	Structural-facial zones of the inner parts experience intensive downwarpings at the beginning of the middle stages. During the beginning stage, the intra-geosyncline experiences dislocations associated with increasing central upheavals. In structure tin-bearing regions are located at zones of junction of the anticlinoria with the adjacent synclinoria, along the junctions of the downwarpings with the slightly mobile geoanticlinal formations or within the latter.
		With the later (acid and ultra-acid) phase of the batholith are associated rare metal (earth)-bearing pegmatites and veined deposits of tin, tungsten, arsenic, molybdenum, bismuth and others.		Tin-bearing bodies and regions associated with granitic batholiths are timed to the plutons near-roofed parts, to separate protuberances of the roof (cupolae) and to large blocks of country-rocks included in the granites.
	BEGINNING AND EARLY	Geosynclinal Downwarping: 1. Intensive submergence with accumulations of siliceous carbonates and terrigenous sequence. 2. Submarine effusions of spilitic and keratophyric lava. 3. Intrusions of ultra-basics and grabbroids with accompanying Cr, Pt, Fe, Ti, Ni, and Cu, etc. 4. With a later phase of folding are associated greatly differentiated basic magmas ranging from gabbro-plagio-granites to syenites. In addition to these differentiated rocks are contact deposits of iron, copper and veined gold-ore. 5. In the second half and final phase of this stage there are effusions and sub-volcanic intrusions of andesites, rhyolites, quartz-albite porphyry. Accompanying these intrusions are pyrite and (Cu, Pb, Zn, Au, Ba, and sulfide) mineralizations.	1. Predominant submergence with local upheavals. 2. Intensive folding. 3. Extention and enlargement of zones of abyssal fractures.	Actual concentrations of tin tion is the tin-bearing regions

the geological development of folded regions (continued)

Tin-bearing intrusives	Groups (branches) of tin mineralizations mineralogical and geochemical types of deposits	Metallogenic associations	Economic and refinery importance
Tin-bearing intrusives of type "A": Tin-bearing batholiths of intrusive type "A" which are acid and ultra-acid granites. Typical are alaskitic, leucocratic pegmatites, etc. Differential characteristics are the enrichment of SiO_2 and Al_2O_3 and the impoverishment of Mg and Ca. Na and K play a predominating role. There are abundant concomitant granite porphyries, pegmatites and aplites. Influence on the country rocks: Orogenic activity and the formation of skarns. Sometimes migmatization and granitization are imposed upon the host-rocks.	Batholith Group: 1). Greisen and veined deposits or quartz-cassiterite and wolframite-cassiterite.	Direct and close association with tungsten, bismuth, arsenic, and beryllium. In places associated with gold-bearing regions.	Yields 70% of the world's resources of tin. Especially important as the first source for tin-bearing deposits.
	2). Natrolitic, albitic and other types of tin-bearing pegmatites.	Very close association with beryllium, lithium, tantalum, niobium, cesium, rubidium and [rare earths].	Small tenor with respect to other constituents. A first source for tin deposit
	3). Metasomatic sulfide cassiterite deposits in carbonate rocks.	Arsenic, more rarely lead-zinc, tungsten (scheelite).	Large
Moderately acid granitoids of the early phase of batholithic intrusions (quartz-diorite, granodiorite, adamellite, etc.,).	Tin-bearing deposits with magnetite and sulfides.	Direct association with iron (magnetite), copper, more rarely tungsten (scheelite). Typical of regions of skarnic scheelite, and scheelite-molybdenite deposits	Insignificant

are not found. Disseminated cassiterite in pyrite deposits are found. Practically the sole excep-
of Japan (Ikuno-Akenobe, etc.).

TABLE 2. Outline of the tin-ore deposits formed during

Stage of develop-ment	The general characteristics of the stages (according to Yu. A. Bilibin, with additions)	Type of tectonic movements	The geologic position (emplacement) of tin-bearing regions
Shields (ancient folded structures)			1. Activated parts of the shields with development of late fractures. 2. Tin-bearing deposits are located in the marginal zones of the shields and gravitate to the forward downwarpings of the adjacent geosynclines. 3. Superposed, activated and late downwarpings.

mination in the middle Mesozoic.

The Alpine types are anomalous to this assumption and indicate a marked reduction in tin mineralization. The data available about this group probably is underestimated in that it depreciates the role of Cenozoic tin mineralization. As indirect confirmation of this position, the Cenozoic also plays a leading role in the occurrence of the type of tin mineralization found in the U. S. S. R.

What are the dynamics of the qualitative changes of tin-bearing deposits? In other words, how does the role of separate genetic types of tin concentrations vary throughout geological history?

On the diagram (fig. 1) it is clearly evident that Precambrian tin-bearing deposits are represented entirely by pegmatite concentrations. Tin-bearing pegmatites plus quartz-cassiterite (38 to 40 percent of volume of resources) are found in the lower and middle Paleozoic. The Variscian metallogeny of tin shows a more complex character; here appear the first sulfide-cassiterite concentrations (Tasmania, Cornwall); a part of which, in the general resources, may be estimated as approximately 20 to 23 percent.

In a more absolute sense, the role of sulfide-cassiterite concentrations increases still more (under similar relative numbers) in the Mesozoic, as a whole, from funnel-shaped deposits in carbonate rocks, as exhibited by the deposits of southeast Asia (southern China; Kinta, Malaya; Pinok, Thailand, and other areas). Finally, the Cenozoic is shown to be the epoch of exceptional development of sulfide-cassiterite mineralization; the role of quartz-cassiterite mineralization is extremely diminished, but tin-bearing pegmatites vanish almost completely.

Thus, we do not see evidence confirming the existence of clearly expressed tin-bearing epochs that are equally productive for the entire world. One may speak rather of a more or less even growth of tin-bearing deposits. In addition, with progressive, quantitative increase of the role of industrial tin concentrations in accordance with the geological development of the earth's crust, a trend also may be very clearly seen toward complication of tin mineralization itself. The appearance of new mineral-geochemical types and the appearance of polymetallic-tin, copper-tin, silver-tin, and other similar multicomponent deposits contribute evidence for this complex. Even the Kimmeridgian, to which belongs more than 60 percent of the world's tin resources, may not be considered specifically a tin-bearing epoch. Indeed, this intensive tin-bearing phase is only located within the limits of one, although very large, belt in southeast Asia. Synchronous formations of other regions of the globe in no way stand out in relation to their tin-bearing deposits (fig. 1).

It has long been noticed and emphasized by Smirnov that the principal [control] of tin-bearing deposits of the earth is by regions of the Pacific Ocean ore belt [25]. They show that only 25 percent of world tin resources of tin are connected with the Atlantic Provinces, including the once large, but now almost completely exhausted resources of the Variscian of Europe. Thus, the excess of tin-bearing deposits developed in the near Pacific Ocean zone is very clearly outlined. Because the sharply predominant mass of tin occurs along Pacific Ocean ring (Mesozoic and Cenozoic), it may be asserted with strong evidence that a specific Pacific Oceanic tin-bearing province exists.

The uneven distribution of accessible tin concentration in the earth's crust is shown clearly to be the result of separate, exceptionally rich zones; for example, the Indonesian-Malayan-Burman and the vast Bolivian tin-bearing belts, where more than half of the world's tin resources are concentrated. The

the geological development of folded regions (concluded)

Tin-bearing intrusives	Groups (branches) of tin mineralizations mineralogical and geochemical types of deposits	Metallogenic associations	Economic and refinery importance
Relatively very late and less deep intrusives of tourmaline pegmatoid granites, alaskitic and rapakiwi granites.	1. Tin-bearing pegmatites 2. More rarely, quartzose veins and greisen-deposits of cassiterite in veined granite. 3. Exceptionally rare are tin-bearing skarns and complex sulfido-tourmaline deposits.	Lithium, tantalum, niobium, and rare earths are characteristic. Less clear association with gold and copper. Tungsten not characteristic.	Large scattered deposits. Ten percent of the world's tin-resources mainly located in northeast Nigeria, Belgian Congo and Rhodesia.

problem is, what are the conditions that probably cause similar anomalous and uneven tin distribution? From what has been shown in the proceeding pages there is obviously a clear control of tin by intermediate and late stages of the geosynclinal development. Consequently, the appearance of corresponding stages during the course of geosynclinal development determines, also, the possible origin of tin-ore deposits. However, it is general knowledge in the majority of regions outside the Pacific Ocean Provinces and the Variscian of Europe that, in spite of favorable geological occurrence corresponding to the intermediate and late stages of development, tin bearing either did not occur at all or was very subdued. It is quite clear that the development of intermediate and late stages in a given folded region, is shown to be the essential condition; but, is far from being sufficient for the formation of commercial tin-bearing deposits.

Another detail is curious in several ore regions, tin in industrial concentrations occurs throughout a series of geological periods inclusive from middle Paleozoic to Paleogene (southern part of the Far East.). This "inclusive" character of the appearance of mineralization evidently is one of the properties characteristic of true tin-bearing provinces. In other words, we come unavoidably to the conclusion about the fatal influence on distribution characteristics of the tin-bearing regions; these characteristics are associated with the type of metallogenic province that contains them.

The specificity of the metallogenesis in geosynclinal regions of the Pacific Ocean Ring, especially, is outlined in the works of Smirnov [25] and Yu. A. Bilibin [5, 6] where is emphasized the existence of several different lines of geosynclinal development, and considered essential to bring forward the peculiar, so-called East Asiatic type of mineralization. This type is peculiar to geosynclines with intensive development of ore bearing (predominantly tin bearing) of intermediate and late stages. This East Asiatic type was contrasted with the Ural type having extensive appearances of metallogenesis of early and late stages [5].

In a recently published article, D. S. Kharkevich also emphasizes the unusual development of near Pacific Ocean geosynclines, that are associated in his opinion, with the fact that "their bedding (author: of folded regions.) did not originate in abyssal-fracture zones" [27]. A similar explanation, in view of the content of the work in question, does not appear to be convincing enough. Up to the present, a rational explanation has not been found for the metallogenic specializations (concentration) of different regions and provinces, including tin-bearing areas. The key to solution of this complicated problem is hidden in the following: 1) characteristics of the abyssal foundation; and, 2) character of the region of magmatic nourishment, where tin-bearing intrusives originate along with the specific development of geosynclines.

Evidently it is a correct observation that "abyssal processes represent phenomena, incomparably greater than those disturbances of the surface layers, that we are accustomed to unite in the understanding of tectonics" [9]. Naturally, it is possible to point out only the most general suppositions about processes occurring at these depths. One of the possible variants in the explanation leads to the assumption of primordial geochemical heterogeneity for different parts of the earth's crust.

The concept about the heterogeneity of the earth's crust is developed in the geophysical investigations of P. K. Kropotkin [11], E. V. Pavlovsky [15], and others. In particular, the so-called oceanic thin crust or "Original Platform" assumed to occur at the bottom of the central parts of the Pacific Ocean, has not been subjected to folding from time immemorial and is completely devoid of sial [15]. The second line of evidence is explained by the

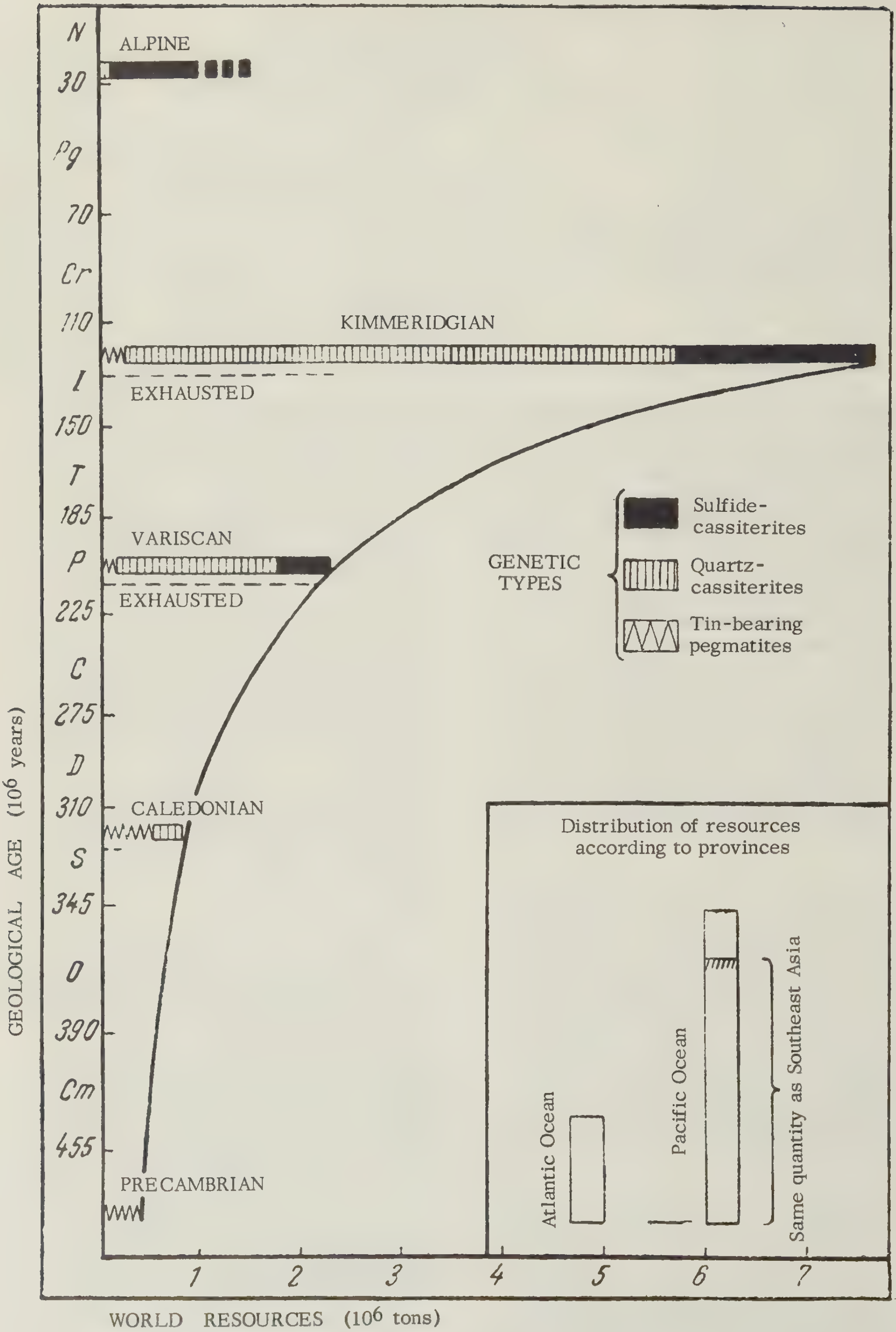

FIGURE 1. Distribution of world tin resources according to geologic time and tin-producing areas.

well-substantiated data of abyssal geophysics and the concentric-zonal physical and geochemical heterogeneity of the globe.

Evidently, the development of a potential tin-bearing province may occur if long-extent abyssal fractures in corresponding stages of geosynclinal development will interact with relatively enriched tin zones of the sial, where will be located chambers of melting substrata and nourishment of magma as ore material. The unique character of the Pacific Ocean Province may be a result of the fact that very large zones of abyssal fractures, developed there over long periods of time, dip gently towards and under the continent. These fractures have been indicated by geophysical methods. Seismic data shows that these imbricated abyssal fractures extend to depths of hundreds of kilometers. Their tremendous importance to the tectonic development of regions bordering the Pacific Ocean is further confirmed in the works of A. N. Zavaritsky [9].

In connection with this, increasing geological-geophysical study of zones of deep fractures that control the distribution of tin-bearing bodies and regions appears to be an urgent task; especially in the Far East, where several of these fractures have been tentatively mapped.

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REFLECTION AND REFRACTION OF ELASTIC WAVES, GENERAL THEORY OF BOUNDARY RAYLEIGH WAVES[1]

by V. G. Gogoladze

• *translated by Dr. I. I. Peters (Part I) and Dr. H. P. Thielman (Part II)* •

ABSTRACT

The problem of reflection and refraction of elastic waves in different media getting in touch along a plane is solved in this work. The problem has received complete solution owing to an investigation of the complex nonstationary waves and boundary Rayleigh waves, which arise with reflection and refraction of elastic waves. The investigation is based upon an examination of the roots of some function of a complex variable over the total plane of the latter. In result of a complete investigation of the properties of this function are determined laws of reflection, refraction and appearance of Rayleigh waves.

In the work are given such conditions which are necessary and sufficient for the appearance of Rayleigh waves at different disturbances. - - Author.

INTRODUCTION

The problem of reflection and refraction of elastic waves has served as a source, from earliest times, for a study of the dynamic theory of elasticity. In the papers of G. Green [1], S. D. Poisson [2], G. Stokes [3], J. MacCullagh [4], J. Rayleigh [5], W. Kelvin [6], and others [7]; different types of particular problems have been considered.

From these studies, laws have been established on the reflection and refraction of longitudinal and transverse waves for the case when the angle of incidence does not exceed the critical angle for full internal reflection. In addition, by using exponential functions with complex numbers, the problem of the reflection and refraction of plane sinusoidal waves was determined for the case in which the angle of incidence of the wave was greater than the critical angle for full internal reflection.

During these studies it appeared, with the use of exponential functions, that one may obtain the reflection and refraction of homogeneous (real) and nonhomogeneous (complex) plane waves.

Utilization of these types of exponential functions with complex linear combinations of the coordinates x, y, and z, and time t, enabled J. Rayleigh [5] to define (1885) a new type of wave in a half-space of certain elastic media.

In addition to the incident and reflected waves, it appears that there exists another wave type; this is a wave propagated along the boundary of the half space and decreases exponentially away from the surface. If one ignores the amplitudes and considers only the vibration phase, then the constant-phase planes travel perpendicularly to the boundary of the half-space with a velocity less than that of the transverse wave in that medium. This velocity does not depend on wave length or frequency, it is dependent only upon the elastic properties of the medium. The complete solution of the problem of elastic-wave propagation in a half space is given in the papers of H. Lamb [8], Academicians V. I. Smirnov and S. L. Sobolev [9], E. A. Naryshkina [10], and E. Hallen [11].

In the works of E. Wiechert and K. Zoeppritz [12], C. Knott [13], R. Stoneley [14], H. Jeffreys [15], K. Sezewa [16], H. Blüt [17], H. Berlage [18], M. Muskat [19], and others [7], devoted to the reflection and refraction of elastic waves in different media in plant contact, two types of problems are examined:

1) Plane waves traveling along prescribed paths, for which determination of the problem is simplified; and,

2) elastic media, in contact with each other, selecting special cases; several numerical examples are examined, (cases of incompressibility, Poisson, etc).

In the fundamental work of Green [1], in particular, it is supposed that the elastic media existing in contact have similar elastic properties but different densities. Based on these studies, the question arises concerning how waves are propagated along such interfaces.

The solution of elastic-wave reflection and refraction in the general case, leads to the study of roots of certain functions of complex variables. The properties of this function determine the nature of the reflected and refracted waves; and, similarly, the origin of Rayleigh surface (boundary) waves. This

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[1] Translated from Otrazheniye prelomleniye uprugikh voln obshchaya teoriya granichnykh voln releya: Akademiya Nauk SSSR, Trudy Seysmologicheskogo Instituta, Moscow-Leningrad, 1947, p. 1-42.

function will be called the Rayleigh function.

For the present study, we present a complete investigation of the Rayleigh function and by this method determine the solution of the problem of elastic-wave reflection and refraction for various arbitrary elastic layers that are in contact. In addition, we also study the necessary and sufficient conditions for the existence of Rayleigh boundary waves.

In Chapter I we present the solution of the problem of reflection and refraction for plane non-stationary waves. Propagation of the disturbance in elastic media is considered to be a summation of plane waves traveling with different velocities, wherein each of these waves is reflected and refracted at the interface with real or complex (imaginary) angles (G. Green). We study in detail the kinematic and dynamic propagation of the disturbance; i. e., the question of elastic optics and, as well, the characteristics of energy flow on nonstationary nonhomogeneous (complex) plane waves.

In Chapter II we investigate the Rayleigh functions. We present the basic theory which shows that the Rayleigh function does not have complex roots and may have only two real roots, corresponding to symmetrical modes with respect to the origin or coordinates, that depend upon the propagation velocity of the Rayleigh surface waves. This velocity is less than the velocities of all four body waves in the two media. We establish the necessary and sufficient conditions for the existence of Rayleigh surface (boundary) waves. Criteria for the existence, or absence, of this wave we express by simple ratios of elastic properties and densities of the two media. From these studies of Rayleigh functions, we develop a theory of the Rayleigh surface (boundary) wave, and give the general formulas for reflection and refraction of nonstationary plane waves.

I. REFLECTION AND REFRACTION OF PLANE ELASTIC WAVES

The Problem of Plane Dynamics in the Propagation of Elastic Vibration

We examine two elastic half-spaces in contact along plane OXZ. The axis OY is perpendicular to the interface. Let medium one $(y > 0)$ have the Lamé elastic constants λ_1, μ_1, and density ρ_1. In the second region $(y < 0)$ the material has Lamé constants λ_2, μ_2, and density ρ_2. Assuming the waves to be independent of the Z direction, we have the problem of plane elastic-waves propagation.

Elastic oscillations are vibrating in the plane OXY and are propagated in the direction OZ; the latter appears as a transverse vibration. The problem of reflection and refraction of such a wave can be computed in an elementary fashion and we shall not present it here.

Particle displacement in the plane OXY is given by (1)

$$u_j = \frac{\partial \varphi_j}{\partial x} + \frac{\partial \psi_j}{\partial y}, \qquad v_j = \frac{\partial \varphi_j}{\partial y} - \frac{\partial \psi_j}{\partial x}, \tag{1}$$

where φ_j and ψ_j are the potentials for longitudinal and transverse waves, which satisfy the following equations

$$\frac{\partial^2 \varphi_j}{\partial x^2} + \frac{\partial^2 \varphi_j}{\partial y^2} = \frac{1}{a^2_j} \frac{\partial^2 \varphi_j}{\partial t^2}, \qquad \frac{\partial^2 \psi_j}{\partial x^2} + \frac{\partial^2 \psi_j}{\partial y^2} = \frac{1}{b^2_j} \frac{\partial^2 \psi_j}{\partial t^2}. \tag{2}$$

$j = 1$ for $y > 0$, $j = 2$ — for $y < 0$,

$a_j = \sqrt{\frac{\lambda_j + 2\mu_j}{\rho_j}}$ — the longitudinal-wave velocity, $b_j = \sqrt{\frac{\mu_j}{\rho_j}}$ — the transverse-wave velocity.

All of the possible combinations of velocities a_1, a_2, b_1, b_2, satisfy one of the following conditions

$$\begin{aligned} &b_2 < b_1 < a_1 < a_2, \quad && b_1 < b_2 < a_1 < a_2, \\ &b_2 < b_1 < a_2 < a_1, && b_1 < b_2 < a_2 < a_1, \\ &b_2 < a_2 < b_1 < a_1. && b_1 < a_1 < b_2 < a_2. \end{aligned} \tag{3}$$

At the interface between the two media $(y = 0)$, in the absence of actual disturbances, the displacement and stress vectors are equal. These conditions in terms of potentials φ_1, ψ_1, φ_2 and ψ_2, can be written as follows.

$$\left(\frac{a_1^2}{b_1^2}-2\right)\Delta\varphi_1+2\frac{\partial^2\varphi_1}{\partial y^2}-2\frac{\partial^2\psi_1}{\partial x\partial y}-\sigma\left[\left(\frac{a_2^2}{b_2^2}-2\right)\Delta\varphi_2+2\frac{\partial^2\varphi_2}{\partial y^2}-2\frac{\partial^2\psi_2}{\partial x\partial y}\right]=0.$$

$$2\frac{\partial^2\varphi_1}{\partial x\partial y}+\frac{\partial^2\psi_1}{\partial y^2}-\frac{\partial^2\psi_1}{\partial x^2}-\sigma\left[2\frac{\partial^2\varphi_2}{\partial x\partial y}+\frac{\partial^2\psi_2}{\partial y^2}-\frac{\partial^2\psi_2}{\partial x^2}\right]=0,$$

$$\frac{\partial\varphi_1}{\partial x}+\frac{\partial\psi_1}{\partial y}-\left[\frac{\partial\varphi_2}{\partial x}+\frac{\partial\psi_2}{\partial y}\right]=0, \qquad (4)$$

$$\frac{\partial\varphi_1}{\partial y}-\frac{\partial\psi_1}{\partial x}-\left[\frac{\partial\varphi_2}{\partial y}-\frac{\partial\psi_2}{\partial x}\right]=0, \qquad y=0$$

where $\sigma=\frac{\mu_2}{\mu_1}$ — is the ratio for rigidity moduli is the first and second media.

The problem of elastic-wave reflection and refraction is based on integrating the system of waves in (2), and the boundary conditions of (4).

Equations (2) have frequently afforded solutions for plane waves (D'Alembert). By plane waves we understand the problem involving wave equations to be:

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}, \qquad (5)$$

in which

$$\Phi=\Phi\,(pt+\alpha x+\beta y), \qquad (6)$$

where

$$\alpha^2+\beta^2=\frac{p^2}{c^2}. \qquad (7)$$

Plane homogeneous (real) waves (6) on each family of plane surfaces (wave fronts), assume the conventionally three dimensional form in the wave-front formula:

$$\alpha x+\beta y+pt=\text{const}. \qquad (8)$$

The problem of reflection and refraction consists of determining the reflected and refracted waves for a given, incident wave. Determination of this problem is elementary for the case in which the disturbance is a homogeneous (real) wave, that satisfies (2) and boundary conditions for homogeneous waves. But this is possible only in special cases. In the general case, we are concerned with the origin of nonhomogeneous (complex) plane waves and Rayleigh surface (boundary) waves.

By way of integreation of reflected and refracted waves, for all significantly permissible angles of incidence ρ, we obtain a wide variety of vibrations that permit determination of the dynamic problem of propagation resulting from arbitrary sources of disturbance.

Reflection and Refraction of Longitudinal and Transverse Waves

Let us assume, for the determination of velocity of propagation of longitudinal and transverse waves in the upper medium ($y > 0$) and in lower medium ($y < 0$), that the following conditions are satisfied;

$$b_1 < b_2 < a_2 < a_1. \qquad (9)$$

Let us assume that in the upper medium ($y > 0$) a plane longitudinal wave is incident at the interface ($y = 0$), having a potential

$$\varphi=\Phi\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\right), \qquad (10)$$

where

$$\frac{p^2}{a_1^2}-\alpha^2>0. \qquad (10')$$

From (1), displacement of the longitudinal wave normal to the wavefront is

$$pt+\alpha x+y\sqrt{\frac{p^2}{a_1^2}-\alpha^2}=\text{const}, \qquad (11)$$

where

$$\operatorname{tg} el=\frac{\alpha}{\sqrt{\frac{p^2}{a_1^2}-\alpha^2}}, \quad \sin el=\frac{a_1\alpha}{p}, \qquad (12)$$

(el is the angle of incidence for the longitudinal wave).

In the upper medium, according to boundary conditions (4), for the incident wave (10), we add the reflected longitudinal and transverse waves as

$$\varphi+\varphi_1=\Phi\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\right)+$$

$$+\Phi_1\left(pt+\alpha x-y\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\right), \qquad (13)$$

$$\psi_1=\Psi_1\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right), y>0$$

Φ_1 and ψ_1 – represent the unknown potential for longitudinal and transverse reflected plane waves,

wherein

$$\operatorname{tg} rl = \frac{\alpha}{\sqrt{\frac{p^2}{a_1^2} - \alpha^2}},$$

$$\sin rl = \frac{a_1\alpha}{p} \tag{14}$$

(rl — is the reflection angle of the longitudinal wave,

$$\operatorname{tg} rt = \frac{\alpha}{\sqrt{\frac{p^2}{b^2} - \alpha^2}}, \quad \sin rt = \frac{b_1\alpha}{p} \tag{15}$$

(rt — is the reflection angle of the transverse wave). (Fig. 1).

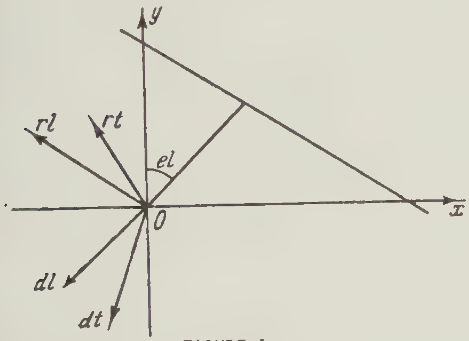

FIGURE 1.

In the lower medium ($y < 0$) the refracted longitudinal and transverse waves are propagated with the potentials

$$\varphi_2 = \Phi_2\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\right),$$

$$\psi_2 = \Psi_2\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right), \; y < 0 \tag{16}$$

where

$$\operatorname{tg} dl = \frac{\alpha}{\sqrt{\frac{p^2}{a_2^2} - \alpha^2}}, \quad \sin dl = \frac{a_2\alpha}{p} \tag{17}$$

(dl is the refraction angle of the compressional wave),

$$\operatorname{tg} dt = \frac{\alpha}{\sqrt{\frac{p^2}{b_2^2} - \alpha^2}}, \qquad \sin t = \frac{b_2\alpha}{p} \tag{18}$$

(dt is the refraction angle of the transverse wave)

In identifying the angles, the first letter indicates: incidence e, reflection r, and refraction d; the second letter indicates wave type: longitudinal wave l, and transverse wave t.

From formulas (14), (15), (17), and (18) we obtain Snell's Law:

$$\frac{\alpha}{p} = \frac{\sin el}{a_1} = \frac{\sin rl}{b_1} = \frac{\sin dl}{a_2} = \frac{\sin dt}{b_2}. \tag{19}$$

To determine the potentials Φ_1, Ψ_1, Φ_2 and Ψ_2 we use the expressions $\varphi + \varphi_1$, ψ_1 (13) and φ_2, ψ_2 (16) with the included boundary conditions (4). We obtain the system of equations:

$$\begin{aligned}
&\left(\frac{p^2}{b_1^2} - 2\alpha^2\right)\Phi_1'' + 2\alpha\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\,\Psi_1'' - \sigma\left[\left(\frac{p^2}{b_2^2} - 2\alpha^2\right)\Phi_2'' - \right.\\
&\qquad \left. - 2\alpha\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\,\Psi_2''\right] = -\left(\frac{p^2}{b_2^2} - 2\alpha^2\right)\Phi'',\\
&-2\alpha\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\,\Phi_1'' + \left(\frac{p^2}{b_1^2} - 2\alpha^2\right)\Psi_1'' - \sigma\left[2\alpha\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\,\Phi_2'' + \right.\\
&\qquad \left. + \left(\frac{p^2}{b_2^2} - 2\alpha^2\right)\Psi_2''\right] = -2\alpha\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\,\Phi'',\\
&\alpha\Phi_1'' - \sqrt{\frac{p^2}{b_1^2} - \alpha^2}\,\Psi_1'' - \left[\alpha\Phi_2' + \sqrt{\frac{p^2}{b_2^2} - \alpha^2}\,\Psi_2''\right] = -\alpha\Phi'', \qquad (20)\\
&-\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\,\Phi_1'' - \alpha\Psi_1'' - \\
&\qquad -\left[\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\,\Phi_2'' - \alpha\Psi_2''\right] = -\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\,\Phi''.
\end{aligned}$$

Solutions of the system (20) in terms of the Φ_1'', Ψ_1'', Φ_2'' and Ψ_2'', are obtained in the following form

$$
\begin{aligned}
\Phi_1'' &= \frac{d_1(p, \alpha)}{R(p, \alpha)} \Phi'' = D_1\Phi'', \\
\Psi_1'' &= \frac{d_2(p, \alpha)}{R(p, \alpha)} \Phi'' = D_2\Phi'', \\
\Phi_2'' &= \frac{d_3(p, \alpha)}{R(p, \alpha)} \Phi'' = D_3\Phi'', \\
\Psi_2'' &= \frac{d_4(p, \alpha)}{R(p, \alpha)} \Phi'' = D_4\Phi'',
\end{aligned}
\tag{21}
$$

where d_1, d_2, d_3, d_4 and R are given by:

$$
\begin{aligned}
d_1 = &-\left[p^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\alpha^2\right]\alpha^2 + \\
&+\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p^2}{b_1^2}-\alpha_2}\left(\frac{\sigma p^2}{b_2^2}+2(1-\sigma)\alpha^2\right)^2 - \\
&-\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\left(\frac{p^2}{b_1^2}-2(1-\sigma)\alpha^2\right)^2 + \\
&+\left(\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p_2}{b_2^2}-\alpha^2}-\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\frac{\sigma p^4}{b_1^2 b_2^2} + \\
&+4(1-\sigma)^2\alpha^2\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2};
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
d_2 = &-2\alpha\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\left\{\left[p^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\alpha^2\right]\left[\frac{\sigma p^2}{b_2^2}+2(1-\sigma)\alpha^2\right] + \right. \\
&\left. +2(1-\sigma)\left(\frac{p^2}{b_1^2}-2(1-\sigma)\alpha^2\right)\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right\}; \\
d_3 = &+\frac{2p^2}{b_1^2}\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\left\{\left(\frac{\sigma p^2}{b_2^2}+2(1-\sigma)\alpha^2\right)\sqrt{\frac{p^2}{b_1^2}-\alpha^2} + \right. \\
&\left. +\left(\frac{p^2}{b_1^2}-2(1-\sigma)\alpha^2\right)\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right\}; \\
d_4 = &-\frac{2\alpha p^2}{b_1^2}\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\left\{\left[p^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\alpha^2\right] - \right. \\
&\left. -2(1-\sigma)\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_1^3}-\alpha^2}\right\};
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
R = &\left[p^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\alpha^2\right]^2\alpha^2 + \\
&+\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\left(\frac{\sigma p^2}{b_2^2}+2(1-\sigma)\alpha^2\right)^2 + \\
&+\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\left(\frac{p^2}{b_1^2}-2(1-\sigma)\alpha^2\right)^2 + \\
&+\left(\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}+\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\frac{\sigma p^4}{b_1^2 b_2^2} + \\
&+4(1-\sigma)^2\alpha^2\sqrt{\left(\frac{p^2}{a_1^2}-\alpha^2\right)\left(\frac{p^2}{b_1^2}-\alpha^2\right)\left(\frac{p^2}{a_2^2}-\alpha^2\right)\left(\frac{p^2}{b_2^2}-\alpha^2\right)}.
\end{aligned}
$$

As in theoretical problems of elasticity, if one disregards rigid motion, then the potentials of elastic disturbance in the upper and lower media may be described by

$$\varphi_1 = \Phi\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right) + $$
$$+ D_1\Phi\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right), \tag{24}$$
$$\psi_1 = D_2\Phi\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right),$$
$$\varphi_2 = D_3\Phi\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\right),$$
$$\psi_2 = D_4\Phi\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right).$$

constants (coefficients of intensity) D_1, D_2, D_3, and D_4 from equations (10), (22), and (23), represent real quantities, and the expressions

$$\sqrt{\frac{p^2}{a_1^2} - \alpha^2}, \sqrt{\frac{p^2}{b_1^2} - \alpha^2},$$
$$\sqrt{\frac{p^2}{a_2^2} - \alpha^2}, \sqrt{\frac{p^2}{b_2^2} - \alpha^2}$$

are real. Therefore, the ground disturbances in an elastic medium for this case are represented by homogeneous (real) plane waves. Equation (12) in this case determines only the real angles of reflected and refracted waves.

We examine now another set of velocity distributions

$$b_1 < b_2 < a_1 < a_2. \tag{25}$$

on the basis of Snell's Law (19) we have the relation

$$\sin dl = \frac{a_2}{a_1} \sin el, \tag{26}$$

where

$$\frac{a_2}{a_1} > 1. \tag{26'}$$

Accordingly, for the variation of the angle of incidence el in the internal $\left(\arc \sin \frac{a_1}{a_2}, \frac{\pi}{2}\right)$ the angle of refraction dl of the longitudinal wave is imaginary. Consequently, the refracted longitudinal disturbances in the region $y < 0$ may not be regarded as homogeneous (real) plane waves. For determination of the problem of reflected and refracted waves in the general case, we examine these nonhomogeneous plane waves.

First, we study the periodic nonhomogeneous waves. Assume that the periodic wave with potential

$$\varphi = A \cos\left[pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right] \tag{27}$$

is incident on the boundary $y = 0$ at an angle greater than the angle of complete internal reflection; i. e., assume the following relation to be satisfied

$$\frac{p^2}{a_2^2} - \alpha^2 < 0. \tag{28}$$

We express the potential φ (27) as the sum of exponential functions:

$$\varphi = \frac{1}{2}\left[Ae^{i\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right)} + \right.$$
$$\left. + Ae^{-i\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right)}\right]. \tag{29}$$

We determine the vibration resulting from the first term

$$f_1 = \frac{A}{2} e^{i\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right)}. \tag{30}$$

The potentials of reflected and refracted waves may be written as

$$\varphi_1 = G_1 \frac{A}{2} e^{i\left[pt + \alpha x - y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right]},$$
$$\psi_1 = G_2 \frac{A}{2} e^{i\left[pt + \alpha x - y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right]}, \tag{31}$$
$$\varphi_2 = G_3 \frac{A}{2} e^{i\left[pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\right]}, \quad y<0$$
$$\psi_2 = G_4 \frac{A}{2} e^{i\left[pt + \alpha x + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right]}.$$

Using the expressions for the potentials in (30) and (31) and the boundary conditions (4), we obtain the potentials of reflected and refracted waves in the following forms

$$\varphi_1{}^* = D_1{}^* \frac{A}{2} e^{i\left[pt + \alpha x - y\sqrt{\frac{p^2}{a_1{}^2} - \alpha^2}\right]},$$

$$\psi_1{}^* = D_2{}^* \frac{A}{2} e^{i\left[pt + \alpha x - y\sqrt{\frac{p^2}{q_1{}^2} - \alpha^2}\right]} \quad y > 0$$

$$\varphi_2{}^* = D_3{}^* \frac{A}{2} e^{y\sqrt{\alpha^2 - \frac{p^2}{a_2{}^2}}} e^{i\,[pt + \alpha x]}, \tag{32}$$

$$\psi_2{}^* = D_4{}^* \frac{A}{2} e^{i\left[pt + \alpha x + y\sqrt{\frac{p^2}{b_2{}^2} - \alpha^2}\right]}, \quad y < 0.$$

It is evident that the coefficients D_k^* ($k = 1, 2, 3, 4$) are derived from the expression of D_k (21) by replacing

$$\sqrt{\frac{p^2}{a_2{}^2} - \alpha^2} \quad \text{with} \quad i\sqrt{\alpha^2 - \frac{p^2}{a_2{}^2}}.$$

If we take the form of the potential of the incident wave as the second term in the right member of (29)

$$f_2 = \frac{A}{2} e^{-i\left[pt + \alpha x + y\sqrt{\frac{p^2}{a_1{}^2} - \alpha^2}\right]}, \tag{33}$$

then analogous to the above, we obtain the potentials of reflected and refracted waves;

$$\bar{\varphi}_1{}^* = \bar{D}_1{}^* \frac{A}{2} e^{-i\left[pt + \alpha x - y\sqrt{\frac{p^2}{a_1{}^2} - \alpha^2}\right]},$$

$$\bar{\psi}_1{}^* = \bar{D}_2 \frac{A}{2} e^{-i\left[pt + \alpha x - y\sqrt{\frac{p^2}{b_1{}^2} - \alpha^2}\right]}, \tag{34}$$

$$\bar{\varphi}_2{}^* = \bar{D}_3{}^* \frac{A}{2} e^{-i\left[pt + \alpha x + iy\sqrt{\alpha^2 - \frac{p^2}{a_2{}^2}}\right]},$$

$$\bar{\psi}_2{}^* = \bar{D}_4{}^* \frac{A}{2} e^{-i\left[pt + \alpha x + y\sqrt{\frac{p^2}{b_2{}^2} - \alpha^2}\right]}.$$

The coefficients $\bar{D}_k^*$ are obtained from $\bar{D}_k$ by replacing $\sqrt{\frac{p^2}{a_2{}^2} - \alpha^2}$ by

$$i\sqrt{\alpha^2 - \frac{p^2}{a_2{}^2}}.$$

In this manner, D_k^* and $\bar{D}_k^*$ appear as complex-conjugate quantities. Putting these two parts of the solution together we have

$$\varphi_1 = A\cos\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_2{}^2} - \alpha^2}\right) + \frac{1}{2}\left[D_1{}^*Ae^{i\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_1{}^2} - \alpha^2}\right)} + \bar{D}_1{}^*Ae^{-i\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_1{}^2} - \alpha^2}\right)}\right],$$

$$\psi_1 = \frac{1}{2}\left[D_2{}^*Ae^{i\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1{}^2} - \alpha^2}\right)} + \bar{D}_2{}^*Ae^{i\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1{}^2} - \alpha^2}\right)}\right], \tag{35}$$

$$\varphi_2 = \frac{1}{2}\left[D_3{}^*Ae^{i\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2{}^2}}\right)} + \bar{D}_3{}^*Ae^{-i\left(pt + \alpha x + iy\sqrt{\alpha^2 - \frac{p^2}{a_2{}^2}}\right)}\right],$$

$$\psi_2 = \frac{1}{2}\left[D_4{}^*Ae^{i\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2{}^2} - \alpha^2}\right)} + \bar{D}_4{}^*Ae^{-i\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2{}^2} - \alpha^2}\right)}\right].$$

It is evident that these equations may be expressed more simply:

$$\varphi_1 = Re\left\{Ae^{i\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1{}^2} - \alpha^2}\right)} + D_1{}^*Ae^{i\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_1{}^2} - \alpha^2}\right)}\right\},$$

$$\psi_1 = Re\left\{AD_2{}^*e^{i\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1{}^2} - \alpha^2}\right)}\right\}, \tag{36}$$

$$\varphi_2 = Re\left\{e^{y\sqrt{\alpha^2 - \frac{p^2}{a_2{}^2}}} AD_3{}^*e^{i\,(pt + \alpha x)}\right\},$$

$$\psi_2 = Re\left\{D_4{}^*Ae^{i\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2{}^2} - \alpha^2}\right)}\right\}.$$

As we see, the longitudinal-refracted-wave potential has varying amplitude that diminishes exponentially with depth; in the present case, we have a nonhomogeneous wave which may be considered to be a plane wave with complex directional cosines (it is sometimes described as a complex plane wave).

Let now an arbitrary nonstationary (plane) wave be incident at an angle larger than for complete reflection, have a potential

$$\varphi = \varphi\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right) \quad (37)$$

From elementary complex-variable theory, it is known that if φ represents a real function for all real values of the argument and has a continuous derivative φ', where the second derivative satisfies the condition $|x^2\varphi''(x)| < M$ (M is a constant); then, such a function may be expressed in the form of a half-sum of two functions of a complex variable: $f_1(z)$ and $f_2(z)$. Of these $f_1(z)$ is regular in the upper half-plane of the variable z and has a derivative $f_1'(z)$, within the closed region; $f_2(z)$ is regular in the lower half-plane of variable z, and has a derivative $f_2'(z)$ within the closed region. In addition, the complex conjugate points with respect to the real axis of $f_1(z)$ and $f_2(z)$ take on a complex-conjugate value and the real axis of the real part of functions $f_1(z)$ and $f_2(z)$ is (coincides with) the value of the function φ.

Now, let the potential φ possess the previously mentioned properties. Then (37) may be written

$$\varphi = \frac{1}{2}\left[f_1\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right) + f_2\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right)\right]. \quad (38)$$

We determine the reflection and refraction of the functions f_1 and f_2. Repeating the elementary calculations derived for the solutions of periodic waves, we obtain finally:

$$\varphi_1 = Re\left\{f_1\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right) + D_1{}^* f_1\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right)\right\}, \quad (39)$$

$$\psi_1 = Re\left\{D_2{}^* f_1\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right)\right\},$$

$$\varphi_2 = Re\left\{D_3{}^* f_1\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\right)\right\},$$

$$\psi_2 = Re\left\{D_4{}^* f_1\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right)\right\},$$

where D_k^* ($k = 1, 2, 3, 4$) is obtained from D_k (21) by replacing $\sqrt{\frac{p^2}{a_2^2} - \alpha^2}$ with $i\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}$, in which f_1 is the regular function in the upper-half complex plane.

The formulas obtained have the following physical interpretations:

$$f_1\left(pt + \alpha x \pm y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right),$$

$$f_1\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right),$$

$$f_1\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right)$$

and represent reflected and refracted homogeneous (real) plane waves; the potential φ_2 represents a plane nonhomogeneous (complex) wave; its phase velocity along the axis OY is an imaginary quantity.

In this manner, the disturbance is attenuated with depth where $y < 0$, although the medium in question does not absorb energy. The energy flow in these waves, in contrast to that in homogeneous waves, travels along curved lines (surfaces). Depending upon the nature of the interface and direction of the incident wave, i. e., for phase-velocity values along the OX axis, $c = \frac{\alpha}{p} = \frac{\sin el}{a_1}$ (fig. 2), reflected and refracted

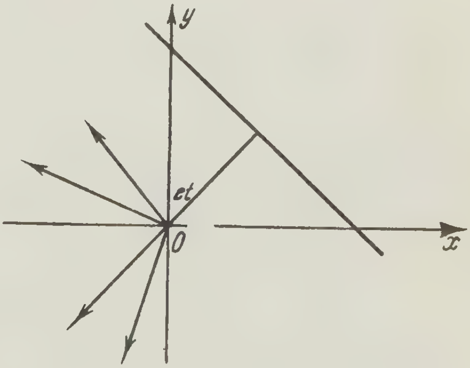

FIGURE 2.

waves can be transformed from homogeneous to nonhomogeneous waves. The later waves obtain their maximum effect at the interface, $y = 0$, attenuating with depth. This wave appears as a generalization of a periodic nonhomogeneous wave. In the problem of vibrations in half-spaces, such waves types originate only from incident transverse waves.

It is readily seen that the analogue of the

solution to this problem applies to the reflection and refraction from incident transverse waves. Its potential may be expressed in the following way.

$$\psi=\frac{1}{2}\left[f_3\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)+\right.$$

$$\left.+f_4\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\right],$$

where

$$\frac{p^2}{b_1^2}-\alpha^2>0 \text{ and } Re f_3=Re f_4=$$

$$=\psi\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right).$$

On the real axis, the function f_3 is regular and f_3' appears as a closed function in the upper half of the complex plane. From the velocity distribution in (9), as in the case of incident longitudinal waves, we determine two particular problems utilizing the functions f_3 and f_4. The final expressions for potentials of the reflected and refracted waves may be written

$$\varphi_1=Re\left\{E_1 f_3\left(pt+\alpha x-y\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\right)\right\},$$

$$\psi_1=Re\left\{f_3\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)+\right.$$

$$\left.+E_2 f_3\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\right\}, \quad (40)$$

$$\varphi_2=Re\left\{E_3 f_3\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\right)\right\},$$

$$\psi_2=Re\left\{E_4 f_3\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)\right\},$$

where the following ranges of velocities are considered

$$b_1<b_2<a_2<a_1<c=\frac{p}{|\alpha|}, \quad (41)$$

Coefficients of intensity E_k are derived similarly to those of D_k and have the following form

$$E_k=\frac{e_k}{R} \qquad (k=1,2,3,4); \qquad (42)$$

The Rayleigh functions R, are derived from equation (23), and e_k has the following values:

$$e_1=+2\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\,\alpha\left\{\left(\frac{p^2}{b_1^2}-\frac{\sigma p^2}{b_2^2}-2(1-\sigma)\alpha^2\right)\left(\frac{\sigma p^2}{b_2^2}+2(1-\sigma)\alpha^2\right)+\right.$$
$$\left.+2(1-\sigma)\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\left(\frac{p^2}{b_1^2}-2(1-\sigma)\alpha^2\right)\right\} \qquad (43)$$

$$e_2=-\left\{\left(\frac{p^2}{b_1^2}-\frac{\sigma p^2}{b_2^2}-2(1-\sigma)\alpha^2\right)^2\alpha^2+\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\left(\frac{\sigma p^2}{b_2^2}+\right.\right.$$
$$\left.+2(1-\sigma)\alpha^2\right)^2-\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\left(\frac{p^2}{b_1^2}-2(1-\sigma)\alpha^2\right)^2+$$
$$+\frac{\sigma p^4}{b_1^2 b_2^2}\left(\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\sqrt{\frac{p^2}{a_2^2}-\alpha^2}-\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)+$$
$$\left.+4(1-\sigma)^2\alpha^2\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right\},$$

$$e_3=\frac{2p^2}{b_1^2}\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\,\alpha\left\{-\left(\frac{p^2}{b_1^2}-\frac{\sigma p^2}{b_2^2}-2(1-\sigma)\alpha^2\right)+\right.$$
$$\left.+2(1-\sigma)\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right\},$$

$$e_4=\frac{2p^2}{b_1^2}\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\left\{\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\left(\frac{\sigma p^2}{b_2^2}+2(1-\sigma)\alpha^2\right)+\right. \qquad (44)$$
$$\left.+\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\left(\frac{p^2}{b_1^2}-2(1-\sigma)\alpha^2\right)\right\}.$$

Provided

$$\frac{p^2}{a_1^2}-\alpha^2<0, \quad \frac{p^2}{a_2^2}-\alpha^2>0, \qquad (45)$$

then the solutions may be given in the following forms

$$\varphi_1 = Re\left\{E_1^* f_3\left(pt+\alpha x + iy\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right)\right\},$$

$$\psi_1 = Re\left\{f_3\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)+ E_2^* f_3\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\right\},$$

$$\varphi_2 = Re\left\{E_3^* f_3\left(pt+\alpha x + y\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\right)\right\}, \qquad (46)$$

$$\psi_2 = Re\left\{E_4^* f_3\left(pt+\alpha x + y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)\right\}.$$

Interpretation of the coefficient E_k^* $(k=1,2,3,4)$ is obtained from (43) where $\sqrt{\frac{p^2}{a_1^2}-\alpha^2}$ is replaced by $i\sqrt{\alpha^2-\frac{p^2}{a_1^2}}$.

Let the phase velocity $c=\frac{p}{|\alpha|}$ now satisfy the following in equality

$$b_2<c<a_2,$$

then the potentials φ_1 and φ_2 will give, according to (46), nonhomogeneous waves, and for following inequality

$$b_1<c<b_2$$

the potentials φ_1, φ_2 and ψ_2, will describe nonhomogenous waves; the potential ψ_1 will express homogeneous (real) plane waves. Naturally, the question arises of the possible origin of vibrations for $c<b_1$ and for complex values of c. This question is tied to the problem of existence of Rayleigh boundary waves, playing a fundamental role in the distribution of seismic disturbances within the earth and on its surface.

Energy Flow in Plane Nonhomogeneous Waves

The coefficients of intensity D_k and E_k $(k=1,2,3,4)$ determine the wave energy arriving as reflected and refracted waves. Between them exists a relation determining the character of energy distribution of the incident wave. These relations, established by C. Knott [13] for periodic waves, apply as well to the energy in nonstationary waves. Actually, if the incident longitudinal wave is given by

$$\varphi=\varphi\left(pt+\alpha x+y\sqrt{\frac{p_2}{a_1^2}-\alpha^2}\right), \qquad (47)$$

then because of the boundary conditions (4) and (20), the following relations exist:

$$2\alpha\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\,(1-D_1)+\left(\frac{p^2}{b_1^2}-2\alpha^2\right)D_2 = 2\sigma\alpha\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\,D_3+\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right)D_4,$$

$$-\alpha(1+D_1)+\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\,D_2 = -\alpha D_3-\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\,D_4, \qquad (48)$$

$$\left(\frac{p^2}{b_1^2}-2\alpha^2\right)D_1+2\alpha\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\,D_2 = \sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right)D_3-2\sigma\alpha\sqrt{\frac{p^2}{b_3^2}-\alpha^2}\,D_4,$$

$$-\sqrt{\frac{p^2}{a_1^1}-\alpha^2}\,(1-D_1)+\alpha D_2 = -\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\,D_3+\alpha D_4.$$

Multiplying the first two equations (48) by one another and adding to the equation obtained the product of the third and fourth equations, we obtain a relation between the squares of the coefficients of intensity D_k:

$$\frac{1}{\sqrt{\frac{p^2}{a_1^2}-\alpha^2}}\left\{\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\,D_1^2+\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\,D_2^2+\delta\left(\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\,D_3^2+\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\,D_4^2\right)\right\}=1, \qquad (49)$$

where $\delta=\frac{\rho_2}{\rho_1}$ — the ratio of densities in the two media.

In an analogous manner, one may obtain a relation for determination of the intensity for incident transverse waves at the surface $y = 0$:

$$\frac{1}{\sqrt{\frac{p^2}{b_1^2} - \alpha^2}} \left\{ \sqrt{\frac{p^2}{a_1^2} - \alpha^2}\, E_1^2 + \right.$$
$$+ \sqrt{\frac{p^2}{b_1^2} - \alpha^2}\, E_2^2 + \delta \left(\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\, E_3^2 + \right.$$
$$\left.\left. + \sqrt{\frac{p^2}{b_2^2} - \alpha^2}\, E_4^2 \right) \right\} = 1. \tag{50}$$

The values

$$\frac{p}{\alpha},\ \sqrt{\frac{p^2}{a_1^2} - \alpha^2},\ \sqrt{\frac{p^2}{b_1^2} - \alpha^2},$$
$$\sqrt{\frac{p^2}{a_2^2} - \alpha^2} \text{ and } \sqrt{\frac{p^2}{b_2^2} - \alpha^2}$$

determine the energy propagation direction of the different waves. Among the coefficients D_k and E_k there are certain relationships which express the so-called reciprocity principle of elastic waves.

If one assumes that the motion of the reflected and refracted waves is rotational, then this naturally determines the reconstructed rotational wave. Let us designate by L_k and K_k the coefficients of intensity of reflected and refracted waves for incident longitudinal and transverse waves from the region of $y < 0$ upon the interface $y = 0$. These coefficients are obtained from D_k and E_k by replacing α with $-\alpha$,

$$\sqrt{\frac{p^2}{a_1^2} - \alpha^2} \text{ by } -\sqrt{\frac{p^2}{a_2^2} - \alpha^2},$$
$$\sqrt{\frac{p^2}{b_1^2} - \alpha^2} \text{ by } -\sqrt{\frac{p^2}{b_2^2} - \alpha^2},\ \sigma \text{ by}$$

$\frac{1}{\sigma}$, and b_1 by b_2.

If we invert the process of wave reflection and refraction in the solution for incident longitudinal waves, the rotational motion gives us the initial rotational wave and cancels out the transverse wave in the region $y > 0$; and, similarly, cancels out the longitudinal and compressional waves in the region $y < 0$; in such a manner we obtain four relations:

$$\begin{aligned} D_1D_1 - D_2E_1 + D_3L_3 + D_4K_3 &= 1, \\ -D_1D_2 + D_2E_2 + D_3L_4 + D_4K_4 &= 0, \\ D_1D_3 - D_2E_3 + D_3L_1 + D_4K_1 &= 0, \\ -D_1D_4 + D_2E_4 + D_3L_2 + D_4K_2 &= 0. \end{aligned} \tag{51}$$

Analogous operations on rotational waves, arising from incident transverse waves, give the relations

$$\begin{aligned} E_1D_1 - E_2E_1 + E_3L_3 + E_4K_3 &= 0, \\ E_1D_2 + E_2E_2 + E_3L_4 + E_4K_4 &= 1, \\ E_1D_3 - E_2E_3 + E_3L_1 + E_4K_1 &= 0, \\ -E_1D_4 + E_2E_4 + E_2L_2 + E_4K_2 &= 0. \end{aligned} \tag{52}$$

In particular for a half-space, we have

$$D_1^2 - D_2E_1 = 1,$$
$$D_1 = E_2.$$

Equations (51) and (52) express the reciprocity (reversibility) principle of elastic waves. The accuracy of relations (51) and (52), for cases in which reflected and refracted waves are homogeneous (real), is verified by direct substitution of the coefficients D_k, E_k, L_k and K_k. These formulas apply to the energy and to the case of nonhomogeneous-wave generation. In the case, D_k is replaced by D_k^*, by E_k E_k^*, etc. Their accuracy for contacts of like elastic media is proved, as based on the Rayleigh boundary wave theory which we interpret further. Particular cases of this principle were obtained by Stokes [3], but without proof.

We will examine now energy propagation in plane waves. If the reflected and refracted waves are homogeneous, then the energy flow of these waves is in a constant direction, along the plane-wave front, appearing as rays along which the energy is propagated.

If, however, during reflection and refraction, nonhomogeneous waves appear, then geometric elastic optics has no possible use for the analysis of energy flow. In this case we replace the concept of rays by introducing the concept of linear energy flow in elastic waves. Let us assume that in the medium $y < 0$ the resultant refraction appears as a nonhomogeneous longitudinal wave and as a homogeneous traverse wave. The front of the longitudinal nonhomogeneous is determined by:

$$pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}} = 0, \tag{53}$$

or

$$pt + \alpha x = 0, \quad y = 0.$$

The wave field of the longitudinal disturbances in $y < 0$ is determined, according to (39), by the formula

$$\varphi_2 = Re D_3^* \varphi_1 \left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}} \right), \tag{54}$$

where the function $\varphi_1(z)$ — is regular in the upper half of the complex plane. The elementary flow of energy $\vec{S}(S_x, S_y)$ is determined by formula (21).

$$S_x = -\mu_2\left[\frac{\partial u}{\partial t}\tau_{xx} + \frac{\partial v}{\partial t}\tau_{xy}\right],$$
$$S_y = -\mu_2\left[\frac{\partial u}{\partial t}\tau_{xy} + \frac{\partial v}{\partial t}\tau_{yy}\right]. \qquad (55)$$

If we substitute into these formulas the expression for the potential φ_2 (54), then we obtain

$$S_x = -\mu_2\left\{\frac{\partial^2\varphi_2}{\partial x\partial t}\left[\left(\frac{a_2^2}{b_2^2}-2\right)\Delta\varphi_2 + 2\frac{\partial^3\varphi_2}{\partial x^2}\right] + 2\frac{\partial^2\varphi_2}{\partial y\partial t}\frac{\partial^2\varphi_2}{\partial x\partial y}\right\} =$$
$$= -\mu_2\left\{Re\left[\left(\frac{p^2}{b_2^2} - \frac{2p^2}{a_2^2} + 2\alpha^2\right)D_3^*\varphi_1''\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a^2}}\right)\right]\right.$$
$$\cdot Re\left[D_3^*\alpha p\varphi_1''\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\right)\right] + \qquad (55')$$
$$+ 2Re\left[-D_3^*\alpha\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\, i\varphi_1''\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\right)\right]\cdot$$
$$\left.\cdot Re\left[-p\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\, iD_3^*\varphi_1''\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\right)\right]\right\}.$$

We introduce the notation

$$D_3^*\varphi_1''\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\right) = \omega_1 + i\omega_2 \qquad (56)$$

and substitute this into (55); taking the real part in (55), we obtain after simplifying the following results:

$$S_x = -\frac{\mu_2\alpha}{b_2^2}\left[\omega_1^2 + 2b_2^2\left(\alpha^2 - \frac{p^2}{a_2^2}\right)(\omega_1^2 + \omega_2^2)\right] \qquad (57)$$

where

$$\alpha^2 - \frac{p^2}{a_2^2} > 0.$$

In an analogous manner we obtain the expression for S_y

$$S_y = -\frac{\mu_2}{b_2^2}\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\,\omega_1\omega_2. \qquad (58)$$

These formulas give rise to an important property of nonhomogeneous-wave propagation. It is evident that the horizontal component of the energy flow S_x has a constant direction; the vertical component may change sign with time and varying location. The energy flow $\vec{S}$ may enter the medium $y < 0$ and exit from it. The overflow (passage) of energy across the nonhomogeneous-wave boundary $y = 0$ produces stresses in different places and at different times. The nonhomogeneous waves, varying in strength according to the uniformity of sign in S_x and variations in S_y, are limited to a narrow zone along the interface between the elastic media $y = 0$, where their effect attains a maximum value. The linear energy flow represents a path of energy flow of nonhomongeneous waves changing with time. The projection of this line on axis OX gives us a fictitious ray, normal to the wave front (53). In the case of periodic optical waves for the Maxwell equations [this phenomenon] is discussed in the fundamental work of A. A. Eykhenvald [21]. The path of linear energy flow is determined by the formula

$$\frac{dy}{dx} = -\frac{\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\,\omega_1\omega_2}{\alpha\left[\omega_1^2 + 2b_2^2\left(\alpha^2 - \frac{p^2}{a_2^2}\right)\left(\omega_1^2 + \omega_2^2\right)\right]}. \qquad (59)$$

In an analogous manner, one may follow the movement of energy of an homogeneous refracted transverse wave. When these waves are homogeneous, we have

$$S_x = -\frac{\mu_2 a}{b_2^2}\omega_3^2,$$
$$S_y = -\frac{\mu_2}{b_2^2}\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\,\omega_3^2, \qquad (60)$$

where

$$\omega_3 + i\omega_4 = D_4^*\varphi\left(pt + \alpha x + \right.$$
$$\left. + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right).$$

The energy paths of these waves coincide with the conventional (normal) plane-wave front. The energy of this wave remains in the medium $y < 0$.

Boundary Waves and the Rayleigh Equation [22]

For a complete study of the problem of wave reflection and refraction, it is necessary to study the Rayleigh boundary waves. We shall look for free vibrations in the elastic medium $(y \gtrless 0)$, for which all longitudinal and traverse disturbances would be nonhomogeneous waves with boundary disturbances. This question is closely tied to the problem of dissipation of seismic radiation along the interface between elastic media. The elastic-disturbance potentials are expressed by using functions of a complex variable whose derivatives approach zero at infinity; i. e., we shall search for oscillatory motion of the type that, in regions far from the interface between media at $y = 0$, will be sufficiently small. The effect of these oscillations will be evident principally in the plane $y = 0$. Such types of oscillations will be called Rayleigh boundary waves.

We shall examine certain possible velocity distributions, for example:

$$b_1 < b_2 < a_2 < a_1. \tag{61}$$

We have studied wave reflection and refraction when the phase velocity $c = \frac{p}{|\alpha|}$ satisfied the inequality $|c| > b_1$. We shall examine the significance of c, finding it cut off at $(-b_1, +b_1)$ and on the complex plane. The potentials φ_1 and ψ_1 in the region $y > 0$ and φ_2, ψ_2 in the region $y < 0$ we write as follows:

$$\left.\begin{aligned} \varphi_1 &= G_1 f_1\left(pt + \alpha x + iy\sqrt{\alpha^2 - \frac{p^2}{a_1^2}}\right), \\ && y > 0 \\ \psi_1 &= G_2 f_1\left(pt + \alpha x + iy\sqrt{\alpha^2 - \frac{p^2}{b_1^2}}\right), \\ \varphi_2 &= G_3 f_1\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\right), \\ && y < 0 \\ \psi_2 &= G_4 f_1\left(pt + \alpha x - iy\sqrt{\alpha^2 - \frac{p^2}{b_2^2}}\right), \end{aligned}\right. \tag{62}$$

where f_1, is a regular function in the upper [complex] half-plane and satisfies the relation $|f_1'(z)| < M$.

The common (joint of composite?) free oscillation of different elastic media satisfy the boundary conditions (4). Substituting the expressions (62) into these conditions, we obtain a system of equations

$$\left.\begin{aligned} &\left\{\left(\frac{p^2}{b_1^2} - 2\alpha^2\right) G_1 - 2\alpha\sqrt{\alpha^2 - \frac{p^2}{b_1^2}}\, iG_2 - \sigma\left(\frac{p^2}{b_2^2} - 2\alpha^2\right) G_3 - \right. \\ &\left. - 2\alpha\sqrt{\alpha^2 - \frac{p^2}{b_2^2}}\, iG_4\right\} f_1'' = 0, \\ &\left\{2\alpha\sqrt{\alpha^2 - \frac{p^2}{a_1^2}}\, iG_1 + \left(\frac{p^2}{b_1^2} - 2\alpha^2\right) G_2 + \right. \\ &\left. + 2\sigma\alpha\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\, iG_3 - \sigma\left(\frac{p^2}{b_2^2} - 2\alpha^2\right) G_4\right\} f_1'' = 0, \\ &\left\{\alpha G_1 + i\sqrt{\alpha^2 - \frac{p^2}{b_1^2}}\, G_2 - \alpha G_3 + i\sqrt{\alpha^2 - \frac{p^2}{b_2^2}}\, G_4\right\} f_1'' = 0, \\ &\left\{i\sqrt{\alpha^2 - \frac{p^2}{a_1^2}}\, G_1 - \alpha G_2 + i\sqrt{\alpha^2 - \frac{p^2}{a_2^2}}\, G_3 + \alpha G_4\right\} f_1'' = 0. \end{aligned}\right. \tag{63}$$

If one disregards rigid motion and uses the arbitrary function f_1, then equation (63) may be written as follows:

$$\left(\frac{p^2}{b_1^2}-2\alpha^2\right)G_1-2\alpha\sqrt{\alpha^2-\frac{p^2}{b_1^2}}\,iG_2-\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right)G_3-$$
$$-2\alpha\sigma\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\,iG_4=0,$$
$$2\alpha\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\,iG_1'+\left(\frac{p^2}{b_1^2}-2\alpha^2\right)G_2+2\sigma\alpha\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\,iG_3-$$
$$-\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right)G_4=0, \tag{64}$$
$$\alpha G_1+i\sqrt{\alpha^2-\frac{p^2}{b_1^2}}\,G_2-\alpha G_3+i\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\,G_4=0,$$
$$i\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\,G_1-\alpha G_2+i\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\,G_3+\alpha G_4=0.$$

In order to determine the homogeneous system equaling zero, we obtain nontrivial solutions for G_1, G_2, G_3 and G_4. The equation for determination of the free-oscillation spectra is as follows:

$$\begin{vmatrix} \frac{p^2}{b_1^2}-2\alpha^2, & -2\alpha\sqrt{\alpha^2-\frac{p^2}{b_1^2}}\,i, & -\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right), & -2\alpha\sigma\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\,i \\ 2\alpha\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\,i, & \frac{p^2}{b_1^2}-2\alpha^2, & 2\sigma\alpha\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\,i, & -\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right) \\ \alpha & i\sqrt{\alpha^2-\frac{p^2}{b_1^2}}, & -\alpha, & i\sqrt{\alpha^2-\frac{p^2}{b_2^2}} \\ i\sqrt{\alpha^2-\frac{p^2}{a_1^2}}, & -\alpha, & i\sqrt{\alpha^2-\frac{p^2}{a_2^2}}, & +\alpha \end{vmatrix}=0. \tag{65}$$

If one examines similarly the functions f_2, , which is regular in the lower complex half-plane the potential will appear as

$$\varphi_1=L_1f_2\left(pt+\alpha x-iy\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right),$$
$$\psi_1=L_2f_2\left(pt+\alpha x-iy\sqrt{\alpha^2-\frac{n^2}{b_1^2}}\right), \tag{66}$$
$$\varphi_2=L_3f_2\left(pt+\alpha x-iy\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\right),$$
$$\psi_2=L_4f_2\left(pt+\alpha x+iy\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\right).$$

The substitution of these expressions in the boundary conditions gives us a system of equations for the determination of coefficients $L_1,\ L_2,\ L_3\quad L_4$.

$$
\begin{gathered}
\left(\frac{p^2}{b_1^2}-2\alpha^2\right)L_1+2\alpha\sqrt{\alpha^2-\frac{p^2}{b_1^2}}\,iL_2-\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right)L_3+\\
+2\sigma\alpha\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\,iL_4=0,\\
-2\alpha\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\,iL_1+\left(\frac{p^2}{b_1^2}-2\alpha^2\right)L_2-2\sigma\alpha\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\,iL_3-\\
-\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right)L_4=0,\\
\alpha L_1-i\sqrt{\alpha^2-\frac{p^2}{b_1^2}}\,L_2-\alpha L_3-i\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\,L_4=0,\\
-i\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\,L_1-\alpha L_2-i\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\,L_3+\alpha L_4=0.
\end{gathered}
\tag{67}
$$

The existence of nontrivial solutions of this system enables us to obtain the equation

$$
\begin{vmatrix}
\frac{p^2}{b_1^2}-2\alpha^2, & 2\alpha\sqrt{\alpha^2-\frac{p^2}{b_1^2}}\,i, & -\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right), & 2\alpha\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\,\sigma i,\\
-2\alpha\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\,i, & \frac{p^2}{b_1^2}-2\alpha^2, & -2\sigma\alpha\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\,i, & -\sigma\left(\frac{p^2}{b_2^2}-2\alpha^2\right);\\
\alpha, & -i\sqrt{\alpha^2-\frac{p^2}{b_1^2}}, & -\alpha, & -i\sqrt{\alpha^2-\frac{p^2}{b_2^2}};\\
-i\sqrt{\alpha^2-\frac{p^2}{a_1^2}}, & -\alpha, & -i\sqrt{\alpha^2-\frac{p^2}{a_2^2}}, & +\alpha.
\end{vmatrix}=0
\tag{68}
$$

In developed form, equations (65) and (68) lead to the equation

$$
\begin{gathered}
\left[p^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\alpha^2\right]^2\alpha^2-\\
-\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\sqrt{\alpha^2-\frac{p^2}{b_1^2}}\left[\frac{\sigma p^2}{b_2^2}+2(1-\sigma)\alpha^2\right]^2-\\
-\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\sqrt{\alpha^2-\frac{p^2}{b_2^2}}\left[\frac{p^2}{b_1^2}-2(1-\sigma)\alpha^2\right]^2-\\
-\left[\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\sqrt{\alpha^2-\frac{p^2}{b_2^2}}+\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\right]\frac{\sigma p^4}{b_1^2b_2^2}+\\
+4(1-\sigma)^2\alpha^2\sqrt{\alpha^2-\frac{p^2}{a_1^2}}\sqrt{\alpha^2-\frac{p^2}{b_1^2}}\sqrt{\alpha^2-\frac{p^2}{a_2^2}}\sqrt{\alpha^2-\frac{p^2}{b_2^2}}=0.
\end{gathered}
\tag{69}
$$

This equation, which determines the free-oscillation spectra, shall be called the Rayleigh equation.

Our problem is concluded with a full investigation of the roots of equation (69) for arbitrary values of the parameters a_1, a_2, b_1, b_2 and σ, characterizing the elastic media. This equation has been solved for certain numerical cases in the works of Stoneley [14] and Sezawa [16].

II. BOUNDARY RAYLEIGH WAVES

Rayleigh Function on a Riemann Surface. [22]

In this chapter we shall study the free-oscillation spectrum and determine necessary and sufficent conditions for the existence of boundary Rayleigh waves in arbitrary elastic media. First of all, we make the substitution

$$\vartheta = \frac{p}{\alpha}. \tag{70}$$

Then Rayleigh's equation (69) takes on the form:

$$\begin{aligned} R(\vartheta) = {} & \left[\vartheta^2\left(\frac{1}{b_1^2} - \frac{\sigma}{b_2^2}\right) - 2(1-\sigma)\right]^2 - \\ & - \left[\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{b_2^2}} + \sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}}\right]\frac{\sigma\vartheta^4}{b_1^2 b_2^2} - \\ & - \sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}}\left[\frac{\sigma\vartheta^2}{b_2^2} + 2(1-\sigma)\right]^2 - \\ & - \sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_2^2}}\left[\frac{\vartheta^2}{b_1^2} - 2(1-\sigma)\right]^2 + \\ & + 4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}}\sqrt{1-\frac{\vartheta^2}{b_2^2}} = 0. \end{aligned} \tag{71}$$

Here $R(\vartheta)$ represents a multiple-valued function of the variable ϑ. In order to make this function more explicit, we construct its Riemann surface. For this purpose we make cuts between the branch points $\pm a_1 \pm a_2$, $\pm b_1 \pm b_2$. For the first sheet of the Riemann surface we choose that one for which the radicals

$$\sqrt{1-\frac{\vartheta^2}{a_1^2}}, \quad \sqrt{1-\frac{\vartheta^2}{b_1^2}}, \quad \sqrt{1-\frac{\vartheta^2}{a_2^2}}, \quad \sqrt{1-\frac{\vartheta^2}{b_2^2}}$$

take the positive sign when $\vartheta = 0$. Thus, the radicals will be positive imaginary quantities for large real values of ϑ. The expansion of the function $R(\vartheta)$ on the first sheet near the origin $\vartheta = 0$, will start with a term of the fourth degree in ϑ; and, $R(\vartheta)$ will take on negative values for small real values of ϑ.

One can establish easily the following approximation, which is of special importance for what follows.

$$\begin{aligned} R(\vartheta) = \vartheta^4 \Bigg[& \left(\frac{1}{a_2^2} - \frac{1}{b_2^2}\right)\left(\frac{1}{a_1^2} + \frac{1}{b_1^2}\right) - 2\sigma\left(\frac{1}{b_1^2 b_2^2} + \frac{1}{a_1^2 a_2^2}\right) + \\ & + \left(\frac{1}{a_1^2} - \frac{1}{b_1^2}\right)\left(\frac{1}{a_2^2} + \frac{1}{b_2^2}\right)\Bigg] + \cdots \end{aligned} \tag{72}$$

Since the velocity of the longitudinal wave is greater than that of the transverse wave, it follows from formula (72) that

$$\frac{1}{a_1^2} - \frac{1}{b_1^2} < 0, \quad \frac{1}{a_2^2} - \frac{1}{b_2^2} < 0; \qquad \sigma = \frac{\mu_2}{\mu_1} > 0.$$

Thus, we have the inequality

$$R(\vartheta) < 0$$

for small real values of ϑ.

Investigation of Rayleigh's Equation [with respect to] Velocity Distribution

$b_2 < b_1 < a_1 < a_2$ for the case where $\sigma = \frac{\mu_2}{\mu_1} > 1$

We draw a semicircle in the lower half-plane of sufficiently large radius. This semicircle with the real axis determines a closed region adequate for study of the roots of Rayleigh's equation. For the purpose of finding these roots, we investigate the changes in the argument of the Rayleigh function $R(\vartheta)$ when ϑ passes once around the boundary of the closed region formed by the real axis and the semicircle of sufficiently large radius (fig. 3).

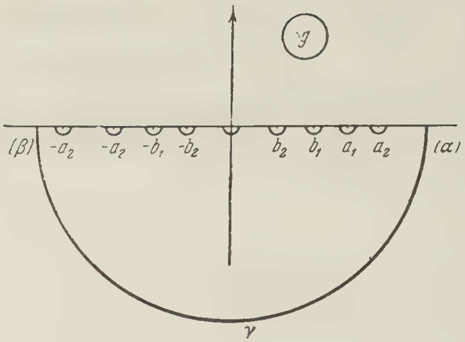

FIGURE 3

On the first sheet of Riemann's surface $R(\vartheta)$ can be expressed in the following form when ϑ has a large positive value of the point (α):

$$
\begin{aligned}
R(\vartheta) = {} & \left[\vartheta^2\left(\frac{1}{b_1^2} - \frac{2}{b_2^2}\right) - 2(1-\sigma)\right]^2 + \\
& + \frac{\sigma\vartheta^4}{b_1^2 b_2^2}\left(\sqrt{\frac{\vartheta^2}{a_1^2} - 1}\sqrt{\frac{\vartheta^2}{b_2^2} - 1} + \sqrt{\frac{\vartheta^2}{a_2^2} - 1}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\right) + \\
& + \sqrt{\frac{\vartheta^2}{a_1^2} - 1}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\left(\frac{\sigma\vartheta^2}{b_2^2} + 2(1-\sigma)\right)^2 + \\
& + \sqrt{\frac{\vartheta^2}{a_2^2} - 1}\sqrt{\frac{\vartheta^2}{b_2^2} - 1}\left(\frac{\vartheta^2}{b_1^2} - 2(1-\sigma)\right)^2 + \\
& + 4(1-\sigma)^2\sqrt{\frac{\vartheta^2}{a_1^2} - 1}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\sqrt{\frac{\vartheta^2}{a_2^2} - 1}\sqrt{\frac{\vartheta^2}{b_2^2} - 1}. \qquad (73)
\end{aligned}
$$

When $\vartheta = \infty$, the function $R(\vartheta)$ is of the order ϑ^6; and, the initial value of its argument is equal to zero.

We shall move along the real axis from right to left. When ϑ changes over the interval (a_2, ∞) the argument of $R(\vartheta)$ is zero; for in this interval [the function] $R(\vartheta)$ is a real quantity.

As soon as we go around the branch point $\vartheta = a_2$, $R(\vartheta)$ becomes a complex quantity; we obtain the following expression for it over the interval (a_1, a_2) :

$$
\begin{aligned}
R(\vartheta) = {} & \left[\vartheta^2\left(\frac{1}{b_1^2} - \frac{\sigma}{b_2^2}\right) - 2(1-\sigma)\right]^2 + \frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{\frac{\vartheta^2}{a^2} - 1}\sqrt{\frac{\vartheta^2}{b_2^2} - 1} + \\
& + \sqrt{\frac{\vartheta^2}{a_1^2} - 1}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\left(\frac{\sigma\vartheta^2}{b_2^2} + 2(1-\sigma)\right)^2 - \\
& - i\left[\frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\sqrt{1 - \frac{\vartheta^2}{a_2^2}} + \right. \\
& + \sqrt{\frac{\vartheta^2}{b_2^2} - 1}\sqrt{1 - \frac{\vartheta^2}{a_2^2}}\left(\frac{\vartheta^2}{b_1^2} - 2(1-\sigma)\right)^2 + \\
& \left. + 4(1-\sigma)^2\sqrt{\frac{\vartheta^2}{a_2^2} - 1}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\sqrt{\frac{\vartheta^2}{b_2^2} - 1}\sqrt{1 - \frac{\vartheta^2}{a_2^2}}\right]. \qquad (74)
\end{aligned}
$$

We see that the argument of the function $R(\vartheta)$ has its values in the lower complex half-plane.

When the variable ϑ passes around the next branch point $v = a_1$ and varies over the interval (b_1, a_1), then Rayleigh's function takes on the form:

$$R(\vartheta) = \left[\vartheta^2\left(\frac{1}{b_1^2} - \frac{\sigma}{b_2^2}\right) - 2(1-\sigma)\right]^2 -$$
$$-4(1-\sigma)^2 \sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\sqrt{\frac{\vartheta^2}{b_2^2}-1} -$$
$$-i\left[\frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\sqrt{1-\frac{\vartheta^2}{a_2^2}} +\right.$$
$$+\sqrt{\frac{\vartheta^2}{b_2^2}-1}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\left(\frac{\vartheta^2}{b_1^2}-2(1-\sigma)\right)^2 +$$
$$+\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_2^2}-1}\,\frac{\sigma\vartheta^4}{b_1^2 b_2^2} +$$
$$\left.+\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\left(\frac{\sigma\vartheta^2}{b_2^2}+2(1-\sigma)^2\right)^2\right]. \qquad (75)$$

In the interval $b_1 < \vartheta < a_1$, the argument of $R(\vartheta)$ changes within the lower complex half-plane, for the imaginary part of $R(\vartheta)$ is negative over this interval.

The passage around the branch point $\vartheta = +b_1$ transfers the variable ϑ into the interval (b_2, b_1), where $R(\vartheta)$ has the expression:

$$R(\vartheta) = \left[\vartheta^2\left(\frac{1}{b_1^2} - \frac{\sigma}{b_2^2}\right) - 2(1-\sigma)\right]^2 - \frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{1-\frac{\vartheta^2}{b_1^2}}\sqrt{1-\frac{\vartheta^2}{a_2^2}} -$$
$$-\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}}\left(\frac{\sigma\vartheta^2}{b_2^2}+2(1-\sigma)\right)^2 +$$
$$+i\left[-4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}} +\right.$$
$$\left.+\left(\frac{\vartheta^2}{b_1^2}-2(1-\sigma)\right)^2\sqrt{1-\frac{\vartheta^2}{a_2^2}} + \frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{1-\frac{\vartheta^2}{a_1^2}}\right]\sqrt{\frac{\vartheta^2}{b_2^2}-1}. \qquad (76)$$

It is important for us to obtain a detailed picture of the properties of the function $R(\vartheta)$ over the interval $b_2 \leqslant \vartheta \leqslant b_1$. For this purpose we introduce the auxiliary functions

$$P(\vartheta) = \left[\vartheta^2\left(\frac{1}{b_1^2} - \frac{\sigma}{b_2^2}\right) - 2(1-\sigma)\right]^2 - \frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}} -$$
$$-\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}}\left(\frac{\sigma\vartheta^2}{b_2^2}+2(1-\sigma)\right)^2. \qquad (77)$$

$$Q(\vartheta) = \left[-4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}} +\right.$$
$$\left.+\sqrt{1-\frac{\vartheta^2}{a_1^2}}\,\frac{\sigma\vartheta^4}{b_1^2 b_2^2} + \sqrt{1-\frac{\vartheta^2}{a_2^2}}\left(\frac{\vartheta^2}{b_1^2}-2(1-\sigma)\right)^2\right]. \qquad (78)$$

In the interval $b_2 < \vartheta < b_1$ the Rayleigh function can be expressed in terms of the function $P(\vartheta)$ and $Q(\vartheta)$ in the following form:

$$R(\vartheta) = P(\vartheta) - iQ(\vartheta)\sqrt{\frac{\vartheta^2}{b_2^2} - 1}. \tag{79}$$

The functions $P(\vartheta)$ and $Q(\vartheta)$ can change their signs as unknown number of times when ϑ changes over the interval (b_2, b_1). This number of changes in sign will depend on the values of the elastic constants.

Only in the case when the ratio of the moduli of rigidity $\sigma = \frac{\mu_2}{\mu_1} > 1$, and $b_2 < b_1$, can one investigate the Rayleigh equation quite simply; for, in this case $Q(\vartheta) > 0$ when $b_2 \leqslant \vartheta \leqslant b_1$. In this case we actually having the following result:

$$Q(\vartheta) = 4(1-\sigma)^2 \sqrt{1 - \frac{\vartheta^2}{a_2^2}} \left(1 - \sqrt{1 - \frac{\vartheta^2}{a_1^2}} \sqrt{1 - \frac{\vartheta^2}{b_1^2}}\right) +$$
$$+ \frac{\vartheta^4}{b_1^4} \sqrt{1 - \frac{\vartheta^2}{a_2^2}} - 4(1-\sigma)\frac{\vartheta^2}{b_1^2}\sqrt{1 - \frac{\vartheta^2}{a_2^2}} + \frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{1 - \frac{\vartheta^2}{a_1^2}}. \tag{80}$$

As the expressions under the radicals in (80) represent proper fractions and because $\sigma > 1$, it follows that $Q(\vartheta) > 0$ over the interval $b_2 \leqslant \vartheta \leqslant b_1$. Therefore, argument $R(\vartheta)$ varies over the lower complex half-plane [of the interval (?)] $b_2 \leqslant \vartheta \leqslant b_1$. If the Rayleigh function is positive at the point $\vartheta = b_2$, then the increase of argument $R(\vartheta)$ will be zero when ϑ varies over the interval (b_2, ∞). If, however, [at the point $\vartheta = b_2$,] $R(b_2) < 0$, then the increase in the argument $R(\vartheta)$ will be $-\pi$ as ϑ varies over [the interval] (b_2, ∞). We shall investigate the changes in the argument $R(\vartheta)$ for two more cases.

$$1)\ R(b_2) > 0, \quad 2)\ R(b_2) < 0. \tag{81}$$

Let us assume first that $R(b_2) > 0$. The Rayleigh function will take on real values in the interval $(0, b_2)$ and will change its sign at least once, for in the neighborhood of $\vartheta = 0$; $R(\vartheta) < 0$, , as was shown above, while at the right end of the interval $(0, b_2)$ $\vartheta = b_2$, we have $R(b_2) > 0$. It follows that the argument $R(\vartheta)$ changes by the amount $-\pi$ over the interval $(0, b_2)$ (if we assume that the sign changed once). When the point $\vartheta = 0$ is circumscribed, then the argument $R(\vartheta)$ changes by the amount -4π, for the point $\vartheta = 0$ is a zero of the fourth order for the Rayleigh function $R(\vartheta)$. The further change in the argument $R(\vartheta)$ along the negative half of the real axis takes place in the symmetrical fashion, as a result of the fact that $R(\vartheta)$ contains quadratics ϑ^2, and, that the radicals enter as even numbers of products. The argument $R(\vartheta)$ changes through $-\pi$ along the negative half of the real axis. We thus obtain the result that the total change in the argument $R(\vartheta)$ as ϑ moves along the real axis from right to left, is $-\pi - 4\pi - \pi = -6\pi$. Going now over to the semicircle of radius $r = \infty$, we recall that here $R(\vartheta)$ is of the order $R(\vartheta) \sim \vartheta^6$. By letting the variable ϑ pass along the semicircle in the lower complex half-plane, we find that argument $R(\vartheta)$ increases by $+6\pi$. The total change in the argument $R(\vartheta)$ is zero [if ϑ varies ?] along the closed contour consisting of the real axis and the lower semicircle. This means that the Rayleigh function has no zero [roots?] in the lower complex half-plane ϑ. Let us assume next that $R(\vartheta)$ has no zero [roots?], $n > 1$, in the interval $(0, b_2)$. Then the total change of the argument $R(\vartheta)$ along the closed contour will be equal to $-2n\pi - 4\pi + 6\pi = 2\pi(1-n)$. Thus, we obtain a negative increase in the argument $R(\vartheta)$, which, of course, is impossible, for this would indicate the presence of poles of the function $R(\vartheta)$ within the semicircle, which cannot happen because of the nature of the Rayleigh function. Thus the hypothesis of the existence of more zeros [roots?] than one in the interval $(0, b_2)$ leads to a contradiction. We see that if the ratio of the moduli of rigidity $\sigma = \frac{\mu_2}{\mu_1} > 1$ and if $R(b_2) > 0$, then the Rayleigh equation has a single root in the interval $(0, b_2)$ and a symmetrically located root in the interval $(-b_2, 0)$.

Next we shall examine the question on the existence of a root under the second condition: $R(b_2) < 0$. At the point $\vartheta = b_1$, we have $\frac{-\pi}{2} < \arg R < 0$. Therefore, the increase in the argument $R(\vartheta)$ is $-\pi$ over the interval (b_2, b_1). It is easy to prove that if ϑ changes further over the interval $(0, b_2)$, then the function $R(\vartheta)$ does not change sign; i. e., the Rayleigh equation does not have a root in the interval $(0, b_2)$, and as a matter of fact, that equation has no root on the first sheet of the Riemann surface. Let us suppose the opposite, namely, let us assume that the Rayleigh function has a zero [root?] on the interval $(0, b_2)$. Because $R(\vartheta)$ takes on negative values at the points $\vartheta = b_2$ and at those near $\vartheta = 0$, it follows that $R(\vartheta)$ will have two zeros [roots?] by our assumption. The change in the argument $R(\vartheta)$ will thus be -2π over the interval $(0, b_2)$. The passage around the point $\vartheta = 0$ would cause the argument $R(\vartheta)$ to change by -4π. Along the negative part of the real axis the change in argument $R(\vartheta)$

will be -3π along the semicircle the increase in argument $R(\vartheta)$ is equal to $+6\pi$. From this it follows that the total change in the argument $R(\vartheta)$ along the closed contour would be -4π. But this is impossible for the function $R(\vartheta)$. It thus follows that there exist no other zeros [roots?] of $R(\vartheta)$, on the real axis than [except trivial roots?] $\vartheta=0$. Because the increase in the argument $R(\vartheta)$ along the closed contour is equal to zero, the function $R(\vartheta)$ has no zeros [roots?] on the first sheet of its Riemann surface. For the case that $\sigma>1$ and $b_2<b_1<a_1<a_2$, a boundary Rayleigh wave can exist if and only if $R(b_2)>0$.

Let us next investigate the zeros [roots?] on $R(\vartheta)$ in case $\sigma<1$, when the signs of the real and imaginary parts of the Rayleigh function can alternate as ϑ varies over the interval (b_2, b_1).

Analysis of the Auxiliary Functions $P(\vartheta)$ and $Q(\vartheta)$ on the Riemann Surface

For the study of the distribution of the zeros [roots ?] of the Rayleigh function in case $\sigma<1$, when the signs of the real and imaginary parts of $R(\vartheta)$ can change [as ϑ varies ?] over $(b_2<\vartheta<b_1)$, we investigate the auxiliary functions $P(\vartheta)$ and $Q(\vartheta)$ in the lower complex half-plane, including the real axis. Near the origin of the coordinate system $(\vartheta=0)$ the function $P(\vartheta)$ has an expansion of the form

$$P(\vartheta)=2(1-\sigma)\left[\frac{1}{a_1^2}-\frac{1}{b_1^2}-\sigma\left(\frac{1}{a_1^2}+\frac{1}{b_1^2}\right)\right]\vartheta^2+\ldots, \tag{82}$$

where $\sigma<1$ and $a_1^2>b_1^2$. For small real values of ϑ, the function $P(\vartheta)$ takes on negative values and has a zero of the second order at $\vartheta=0$. For large values of ϑ, $P(\vartheta)$ is of order ϑ^6. The function $Q(\vartheta)$ has an analogous expansion:

$$Q(\vartheta)=2(1-\sigma)\left[\frac{1}{a_1^2}-\frac{1}{b_1^2}-\sigma\left(\frac{1}{a_1^2}+\frac{1}{b_1^2}\right)\right]\vartheta^2+,\ldots \tag{83}$$

For small real values of ϑ, $Q(\vartheta)<0$.

We shall now prove that the functions $P(\vartheta)$ and $Q(\vartheta)$ can change signs not more than once in the interval $(b_2<\vartheta<b_1)$. For this purpose we investigate the changes in the arguments of these functions over the closed contour consisting of the real axis and the semicircle with large radius in the lower complex half-plane. Let us first examine the function $P(\vartheta)$.

The argument of $P(\vartheta)$ is zero over the interval (a_2,∞). As ϑ passes around the branch point $\vartheta=a_2$, the function $P(\vartheta)$ takes on complex (imaginary) values. In the interval $(a_1<\vartheta<a_2)$ we have

$$P(\vartheta)=\left[\vartheta^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\right]^2+\sqrt{\frac{\vartheta^2}{a_1^2}-1}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\left(\frac{\sigma\vartheta^2}{b_2^2}+\right.$$
$$\left.+2(1-\sigma)\right)^2-\frac{i\sigma\vartheta^4}{b_1^2b_2^2}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}. \tag{84}$$

The argument $P(\vartheta)$ thus changes in the lower complex half-plane. In the interval $b_1<\vartheta<a_1$ we have

$$P(\vartheta)=\left[\vartheta^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\right]^2-i\left[+\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\left(\frac{\sigma\vartheta^2}{b_2^2}+\right.\right.$$
$$\left.\left.+2(1-\sigma)\right)^2+\frac{\sigma\vartheta^4}{b_1^2b_2^2}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\right]. \tag{85}$$

We see that the argument $P(\vartheta)$ changes in the lower complex half-plane. As we pass around the point $\vartheta=b_1$, the function takes on a new form again; in the interval $b_2<\vartheta<b_1$ we have the following expression for $P(\vartheta)$:

$$P(\vartheta)=\left\{\left[\vartheta^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\right]^2-\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}}\left(\frac{\sigma\vartheta^2}{b_1^2b_2^2}+\right.\right.$$
$$\left.\left.+2(1-\sigma)\right)^2-\frac{\sigma\vartheta^4}{b_1^2b_2^2}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}}\right\}. \tag{86}$$

We prove that if ϑ varies over the interval $(b_2 < \vartheta < b_1)$ then the function $P(\vartheta)$ can change its sign not more than once. Let us suppose that one change in sign occurs. Then [at this point $\vartheta = b_2$,] $P(b_2) < 0$, while [at $\vartheta = b_1$ it is positive],

$$P(b_1) = \left[1 - \sigma\left(2 - \frac{b_1^2}{b_2^2}\right)\right]^2 \geqslant 0. \quad (87)$$

As ϑ varies over the interval $(0 < \vartheta < b_2)$, the function $P(\vartheta)$ will either not change sign or it will have at least two zeros [roots ?] as a result of the fact that it is negative near the point $\vartheta = 0$, and $(P(b_2) < 0)$.

Let us consider the first possibility. We assume that $P(b_2) < 0$ and that as ϑ varies over the interval $(0 < \vartheta < b_2)$, the function $P(\vartheta)$ has no zeros [roots ?]. Under all these assumptions, let us compute the increase in the argument of $P(\vartheta)$ as ϑ passes along the closed contour consisting of the real axis and the semicircle. [As ϑ varies?] over the interval $b_2 < \vartheta < \infty$, the argument $P(\vartheta)$ will change by $-\pi$. The passage of ϑ around the point $\vartheta = 0$ will cause the argument $P(\vartheta)$ to change by -2π. Over the interval $(-\infty < \vartheta < b_2)$ the argument $P(\vartheta)$ will change by $-\pi$. [As ϑ passes ?] along the lower semicircle, the argument of $P(\vartheta)$ will gain an increase of 6π. Thus the total change in the argument $P(\vartheta)$ over the closed contour will be $-\pi - 2\pi - \pi + 6\pi = 2\pi$, i. e., there must exist a complex zero [root] of $P(\vartheta)$ in the lower complex half-plane.

Next, let us consider the second possibility. We suppose that $P(\vartheta)$ has two zeros [roots ?] in the interval $(0 < \vartheta < b_2)$, Then the change in the argument $P(\vartheta)$ along the closed contour will be equal to $-\pi - 2\pi - 2\pi - 2\pi - \pi + 6\pi = -2\pi < 0$. We are thus led to a contradiction, for $P(\vartheta)$ does not have poles within the semicircle.

We now suppose that the function $P(\vartheta)$ has two zeros [roots ?] in the interval (b_2, b_1) and that $P(b_2) > 0$. In this case, the function $P(\vartheta)$ will have at least one zero [root ?] in the interval $(0, b_2)$. The change in the argument of $P(\vartheta)$ will take place in the following way. In the interval $0 < \vartheta < \infty$, the change in the argument of $P(\vartheta)$ is equal to -3π, while at the point $\vartheta = 0$, the change in the argument $P(\vartheta)$ is -2π. Along the negative part of the real axis, the argument of $P(\vartheta)$ changes through -3π; while, along the lower semicircle, it increases to 6π. The total change of the argument is equal to

$$-3\pi - 2\pi - 3\pi + 6\pi = -2\pi < 0,$$

that is, we arrive at a contradiction.

If, however, we assume that the number of changes in sign of $P(\vartheta)$ in the interval $0 < \vartheta < b_2$ is greater than one, we again arrive at a contradiction.

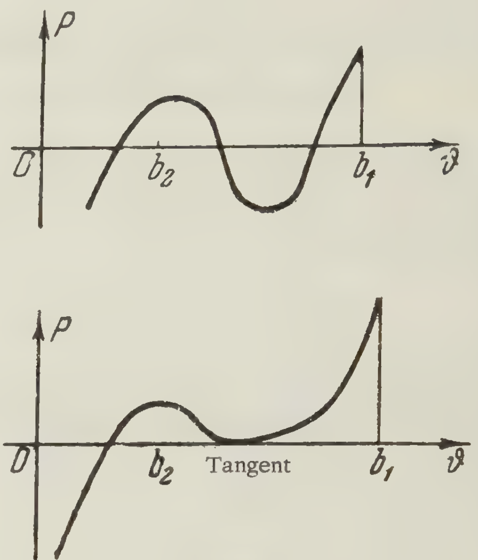

FIGURE 4.

An analogous argument shows that the function $Q(\vartheta)$ also cannot change sign more than once in the interval $b_2 < \vartheta < b_1$. Indeed, at the point $\vartheta = b_1$, we have $Q(b_1) > 0$. In the interval $a_2 < \vartheta < \infty$, the argument of $Q(\vartheta)$ is equal to $\frac{\pi}{2}$.

$$Q(\vartheta) = \left[4(1-\sigma)^2 \sqrt{\frac{\vartheta^2}{a_1^2} - 1} \cdot \sqrt{\frac{\vartheta^2}{a_2^2} - 1} \sqrt{\frac{\vartheta^2}{b_1^2} - 1} + \left(\frac{\vartheta^2}{b_1^2} - 2(1-\sigma)\right)^2 \sqrt{\frac{\vartheta^2}{a_2^2} - 1} + \frac{\sigma\vartheta^4}{b_1^2 b_2^2} \sqrt{\frac{\vartheta^2}{a_1^2} - 1}\right] i. \quad (88)$$

In the internal $a_1 < \vartheta < a_2$ the argument of $Q(\vartheta)$ changes over the interval $\left(0, \frac{\pi}{2}\right)$.

$$Q(\vartheta)=4(1-\sigma)^2\sqrt{\frac{\vartheta^2}{a_1^2}-1}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}+$$
$$+\left(\frac{\vartheta^2}{b_1^2}-2(1-\sigma)\right)^2\sqrt{1-\frac{\vartheta^2}{a_2^2}}+\frac{i\sigma\vartheta^4}{b_1^2\,b_2^2}\sqrt{\frac{\vartheta^2}{a_1^2}-1}. \qquad (89)$$

In the interval $b_1<\vartheta<a_1$, we have the following expression for $Q(\vartheta)$

$$Q(\vartheta)=\frac{\sigma\vartheta^4}{b_1^2\,b_2^2}\sqrt{1-\frac{\vartheta^2}{a_1^2}}+\left[\frac{\vartheta^2}{b_1^2}-2(1-\sigma)\right]^2\sqrt{1-\frac{\vartheta^2}{a_2^2}}-$$
$$-i\,4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\cdot\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}. \qquad (90)$$

The argument of $Q(\vartheta)$ changes over the interval $\left(0,-\frac{\pi}{2}\right)$. The function $Q(\vartheta)$ has the following representation in the interval $b_2<\vartheta<b_1$:

$$Q(\vartheta)=\frac{\sigma\vartheta^4}{b_1^2\,b_2^2}\sqrt{\frac{\vartheta^2}{a_1^2}-1}+\left[\frac{\vartheta^2}{b_1^2}-2(1-\sigma)\right]^2\sqrt{\frac{\vartheta^2}{a_2^2}-1}-$$
$$-4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_2^2}}. \qquad (91)$$

Let us assume that $Q(\vartheta)$ changes sign only once in the interval $b_2<\vartheta<b_1$: Then the function $Q(\vartheta)$ will take on a negative value at the point $\vartheta=b_2$, and we have $Q(b_2)<0$.

We see that the change in the argument of $Q(\vartheta)$ is equal to $-\pi$ (fig. 5). The function

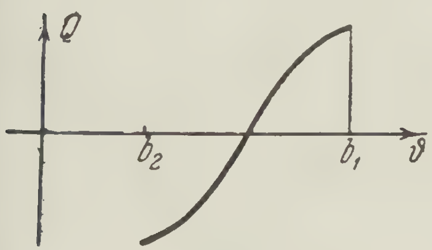

FIGURE 5

$Q(\vartheta)$ cannot change sign in the interval $0<\vartheta<b_2$. A passage around the point $\vartheta=0$ causes a change of -2π in the argument of $Q(\vartheta)$. In the interval $(-\infty<\vartheta<0)$, the argument of $Q(\vartheta)$ changes by $\frac{-3\pi}{2}$, and along the semicircle by $+5\pi$. Thus the total change in argument $Q(\vartheta)$ along the closed contour is equal to

$$-\frac{\pi}{2}-\pi-2\pi-\frac{3\pi}{2}+5\pi=0,$$

That is, $Q(\vartheta)$ has no zero [roots ?] in the lower complex half-plane.

If, however, we suppose that $Q(\vartheta)$ changes sign in the interval $0<\vartheta<b_2$, then the total change in the argument $Q(\vartheta)$ along the closed contour will be negative; i. e., we arrive at a contradiction.

Finally, let up suppose that the function $Q(\vartheta)$ changes sign more than once in the interval (b_2, b_1). Then the change in the argument $Q(\vartheta)$ along the closed contour will be negative (fig. 6):

$$-\frac{\pi}{2}-3\pi-2\pi-3\pi-\frac{\pi}{2}+$$
$$+5\pi=-4\pi<0.$$

This establishes again a contradiction.

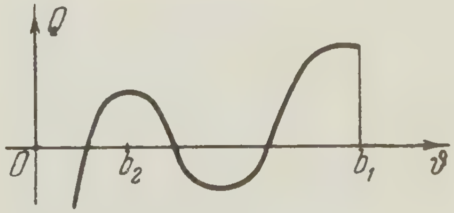

FIGURE 6

Thus, the final result is that the function $P(\vartheta)$ and $Q(\vartheta)$ can have only one root in the interval $b_2<\vartheta<b_1$.

Investigation of the Rayleigh Equation for the Velocity Distribution

$b_2 < b_1 < a_1 < a_2$ and when $\sigma = \frac{\mu_2}{\mu_1} < 1$.

On the basis of the properties established for the function $P(\vartheta)$ and $Q(\vartheta)$ we can assert that the Rayleigh function can have only one zero [root ?] in the interval $(0, b_2)$ if $\sigma < 1$, and if $R(b_2) > 0$. As a result of the fact that $R(b_2) > 0$ and $R(\vartheta) < 0$ for small real values of ϑ, a zero [root ?] $R(\vartheta)$ actually exists. We shall now prove the uniqueness of the root of the equation $R(\vartheta) = 0$. Let us suppose the opposite; then, the equation $R(\vartheta) = 0$ must have at least three roots (fig. 7).

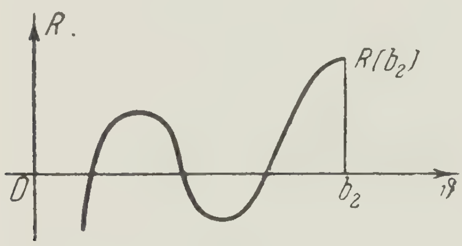

FIGURE 7

When ϑ varies over the interval $(0, b_2)$, the change in the argument of $R(\vartheta)$ is equal to -3π. When ϑ passes around the point $\vartheta = 0$, the argument of $R(\vartheta)$ changes by -4π. Along the negative part of the real axis, the change in the argument [of $R(\vartheta)$] is equal to -3π. Finally, along the lower semicircle, the argument of $R(\vartheta)$ changes by $+6\pi$. The total change in the argument of $R(\vartheta)$ along the closed contour is

$$-3\pi - 4\pi - 3\pi + 6\pi = -4\pi < 0.$$

The change in the argument has turned out to be negative. But this is impossible. This established the uniqueness of the root of the equation $R(\vartheta) = 0$.

We now show that there exists no root of the Rayleigh equation in the lower complex half-plane. We study again the changes in the argument of $R(\vartheta)$. This argument changes by the amount $-\pi$ [as ϑ varies ?] over the interval $0 < \vartheta < b_2$. As ϑ passes around the point $\vartheta = 0$, the change in the argument of $R(\vartheta)$ is -4π. Over the interval $-b_2 < \vartheta < 0$, the argument of $R(\vartheta)$ changes by $-\pi$, while along the semicircle its increase is $+6\pi$. Besides that, the change in the argument $R(\vartheta)$ is zero over the intervals $-\infty < \vartheta < -b_2$, and $b_2 < \vartheta < \infty$. Thus, the total change in the argument of $R(\vartheta)$ is zero [if ϑ varies ?] over the closed contour consisting of the real axis and the lower semicircle.

This established the absence of roots of the Rayleigh equation in the lower complex half-plane.

If the condition $R(b_2) > 0$ is fulfilled, then there exists a single root for the Rayleigh equation.

Necessary and Sufficient Conditions for the Existence of Rayleigh Boundary Waves for the Velocity Distribution $b_2 < b_1 < a_1 < a_2$ and for $\sigma < 1$

Let us suppose that [at point $\vartheta = b_2$] $R(b_2) < 0$. In this case the change in the argument of the Rayleigh function will be $+\pi$ or $-\pi$ as ϑ varies over the interval $b_2 < \vartheta < \infty$. Other possibilities are excluded, for the functions $P(\vartheta)$ and $Q(\vartheta)$ cannot change signs more than once in the interval $b_2 < \vartheta < b_1$, as was established above. At the point $\vartheta = b_1$, the argument of $R(b_1)$ will lie in the interval $\left(0, -\frac{\pi}{2}\right)$. By hypothesis [at point $\vartheta = b_2$] $R(b_2) < 0$; therefore, the change from the value $R(b_1)$ to $R(b_2)$ can occur by passing along arbitrary paths I or II as shown in Figure 8.

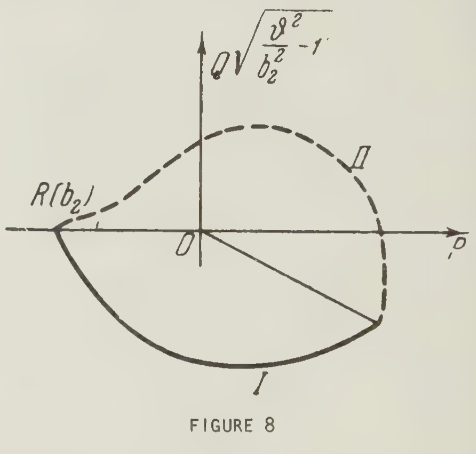

FIGURE 8

We shall prove that the change in the argument of $R(\vartheta)$ is $-\pi$ over the interval $b_2 < \vartheta < b_1$ for arbitrary elastic constants. For this purpose we have to establish the following theorem:

When the variable ϑ changes over an arbitrary interval (b_2, b_1) from $\vartheta = b_1$ to $\vartheta = b_2$, then the function $P(\vartheta)$ will change its sign sooner than the function $Q(\vartheta)$; i. e., for every [value of] ϑ such that $b_2 < \vartheta < b_1$, if $P(\vartheta) = 0$, then $Q(\vartheta) > 0$.

Indeed, if the function $P(\vartheta)$ has the value zero at some point of the interval (b_2, b_1), then the single change in sign of this function will take place at this point. Therefore, if it should happen that $Q(\vartheta) > 0$, then this would

indicate that the real part of the Rayleigh function would change sign sooner than the imaginary part $-Q\ (\vartheta)\sqrt{1-\frac{\vartheta^2}{b_2^2}}$. Thus, the change in the argument of the Rayleigh function $R\ (\vartheta)$ would be equal to $-\pi$. in the interval $b_2 < \vartheta < b_1$.

For the proof of the theorem we shall study the geometric properties of the functions $P\ (\vartheta)$ and $Q\ (\vartheta)$ in the ξ, η, plane, where $\xi = 2\,(1-\sigma) = 2\left(1-\frac{\mu_2}{\mu_1}\right)$, $\eta = \delta = \frac{\rho_2}{\rho_1}$. There exists an obvious relation between σ and δ; namely: $\delta = \sigma\frac{b_1^2}{b_2^2}$, which can be rewritten in terms of the coordinates ξ and η as:

$$\eta = \frac{b_1^2}{b_2^2}\left(1-\frac{\xi}{2}\right), \tag{92}$$

where $\sigma < 1$ and $\frac{b_1^2}{b_2^2} > 1$ (for the velocity distributions $b_2 < b_1 < a_1 < a_2$).

If we make the change of the variable in the Rayleigh function

$$\frac{\vartheta^2}{b_1^2} = \zeta^2, \tag{93}$$

then that function takes the following form:

$$R\ (\zeta) = (\zeta^2\eta+\xi)^2 - 2\,\zeta^2(\zeta^2\eta+\xi) + \zeta^4 - \zeta^4\eta\sqrt{1-\zeta^2}\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}} -$$
$$-\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}\sqrt{1-\zeta^2}\,(\zeta^2\eta+\xi)^2 - i\left[-\xi^2\sqrt{1-\zeta^2}\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}\cdot\right.$$
$$\cdot\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}} + \sqrt{1-\frac{\zeta^2 b_2^2}{a_1^2}}\,\eta\zeta^4 + \sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\,(\zeta^2 -$$
$$\left.-\,\xi)^2\right]\sqrt{\frac{\zeta^2 b_1^2}{b_2^2}-1} = P\,(\zeta) - iQ\ (\zeta)\sqrt{\frac{\zeta^2 b_1^2}{b_2^2}-1}\,.$$

In the ξ, η plane let us consider the curves

$$P\left((\xi,\eta,\zeta,\frac{b_1}{a_1},\frac{b_1}{a_2}\right) = (\zeta^2\eta+\xi)^2 - 2\zeta^2(\zeta^2\eta+\xi) + \zeta^4 -$$
$$-\sqrt{1-\frac{\zeta^2 h_1^2}{a_2^2}}\sqrt{1-\zeta^2}\,\eta\zeta^4 - \sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}\sqrt{1-\zeta^2}\,(\zeta^2\eta+\xi)^2 = 0,$$
$$Q\ \left(\xi,\eta,\zeta,\frac{b_1}{a_1},\frac{b_1}{a_2}\right) = -\xi^2\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\sqrt{1-\zeta^2} +$$
$$+\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}\,\eta\,\zeta^4 + \sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\,(\zeta^2-\xi)^2 = 0. \tag{94}$$

First we examine the curve $Q = 0$. The intersection of this curve with the axis $\eta = 0$ determines two real points:

$$\xi_1 = \frac{\zeta^2}{1+\sqrt[4]{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}}, \qquad \xi_2 = \frac{\zeta^2}{1-\sqrt[4]{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}}. \tag{95}$$

The axis of the parabola $Q = 0$ is parallel to the ordinate axis; and, the parabola opens downward. It has the form shown in Figure 9. The dimensions of this parabola are determined by the numerical

values of the physical constants of the considered elastic media.

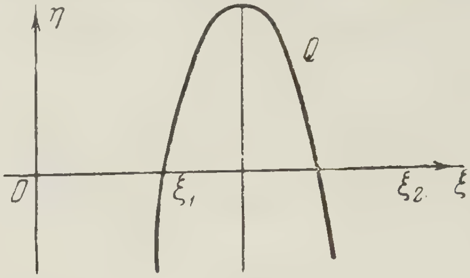

FIGURE 9

Let us next construct the parabola $P = 0$. The points of intersection of this parabola with the $\eta = 0$, are determined by the equation:

$$\xi^2\left(1-\sqrt{(1-\zeta^2)\left(1-\zeta^2\frac{b^2}{a_1^2}\right)}\right)-2\,\zeta^2\xi+\zeta^4=0,$$

which coincides with the equation that determines the points of intersection of the parabola $Q = 0$ with the abscissa axis. [As a result, the parabolas $P = 0$ and $Q = 0$ have two general points of intersection ξ_1, and ξ_2, lying in the abscissa axis. Omitted in translation: Ed.]

Let us now find the points of intersection of the parabola $P = 0$ with the ordinate axis. Setting $\xi = 0$ in the equation of the parabola we find the roots η_1 and η_2 :

$$\eta_1=\frac{2+\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)}+\sqrt{4\left(\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)}+\right.}}{2\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)}\rightarrow$$

$$\rightarrow\frac{\left.+\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}+(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)\right)}{2\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)},\qquad(96)$$

$$\eta_2=\frac{2+\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)}-\sqrt{4\left(\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)}+\right.}}{2\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)}\rightarrow$$

$$\rightarrow\frac{\left.+\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}+(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)\right)}{2\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)}$$

Because $1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}>0$, the enumerator η is positive as is easily verified. We see that the parabola $P = 0$ intersects the ordinate axis in its positive part.

We show next that the parabola $P = 0$, cuts the $O\xi$ axis at an acute angle in the point

$$\xi_1 = \frac{\zeta^2}{1+\sqrt[4]{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}}$$

Let us calculate the value of the derivative η' at the point $\xi = \xi_1$ from the equation $P = 0$. We obtain

$$\eta'_{\xi=\xi_1} = -\frac{2\sqrt[4]{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}}{\zeta^2\left[2\sqrt[4]{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}+\sqrt[4]{\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)(1-\zeta^2)}\right]}. \qquad (97)$$

This determines the direction of the curve $P = 0$ at the point $\xi = \xi_1$.

Next, we prove that the remaining two possible points of intersection of the parabola are real and lie in the lower half-plane, $\eta < 0$. This circumstance makes it possible for us to determine a geometrical relationship between the parabolas $P = 0$ and $Q = 0$.

Let us solve the equations $P = 0$ and $Q = 0$, simultaneously.

From the equation $Q = 0$ we obtain the value of the ordinate

$$\eta = \frac{\left[\xi^2\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}-(\zeta^2-\xi)^2\right]\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}}{\zeta^2\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}} \qquad (98)$$

Substituting this value of η into the equation $P = 0$, we obtain the following relation:

$$\eta\left[\zeta^4\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)\eta+2\xi\zeta^2\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)-\right.$$
$$\left.-\zeta^4\left(2+\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)}+\frac{\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}}{\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}}\right)\right]=0. \qquad (99)$$

This equation is equivalent to the two equations:

1) $\eta = 0$, which determines points that lie on the abscissa axis;

$$2)\ \zeta^4\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)\eta+2\xi\zeta^2\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)-$$
$$-\zeta^4\left(2+\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)}+\frac{\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}}{\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}}\right)=0. \qquad (100)$$

Making the proper substitution in this equation, we obtain, by (98), the following relation for ξ:

$$\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)^2\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)\xi^2-2\xi\zeta^2\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)\cdot \qquad (101)$$
$$\cdot\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\left(\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}+\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\right)+\zeta^4\left(\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}+\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\right)^2=0.$$

The roots of it are

$$\xi_3 = \xi_1 = \frac{\zeta^2\left(\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}+\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\right)}{\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_1^2}\right)}\right)\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}}. \qquad (102)$$

The equality of the roots $\xi_3 = \xi_4$, indicates that the parabolas $P = 0$ and $Q = 0$ are tangent to each other. The ordinate of the point of tangency is, by (102), equal to

$$\eta_3 = \eta_4 = -\frac{\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}-\sqrt{1-\zeta^2}\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}}{\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\left(1-\sqrt{(1-\zeta^2)\left(1-\frac{\zeta^2 b_1^2}{a_2^2}\right)}\right)}. \qquad (103)$$

As $a_1 > b_1$, we have the inequality

$$\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}} > \sqrt{1-\zeta^2}.$$

It is easy to see that

$$\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}-\sqrt{1-\zeta^2}\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}} > \sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}\left(1-\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}}\right). \qquad (104)$$

In cognizance of the fact that

$$\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}} < 1,$$

we have [the inequality]

$$\sqrt{1-\frac{\zeta^2 b_1^2}{a_1^2}}-\sqrt{1-\zeta^2}\sqrt{1-\frac{\zeta^2 b_1^2}{a_2^2}} > 0. \qquad (105)$$

On the basis of (105) we can establish the fact that the ordinates at the points of tangency of the parabolas have negative values.

The preceding investigation permits us to construct the parabolas $P = 0$ and $Q = 0$, and to determine their geometric position with respect to each other. This position is shown in Figure 10.

The parabola $Q = 0$ lies within the parabola $P = 0$ in the region $\eta = \delta = \frac{\rho_2}{\rho_1} > 0$, and the vertex of the parabola $Q = 0$, lies inside the parabola $P = 0$. At the point $(0,0)$ the function P has, by (94), a positive value:

$$P\left(0, 0\zeta, \frac{b_1}{a_2}, \frac{b_1}{a_1}\right) > 0. \qquad (106)$$

The function $\left(\xi, \eta, \zeta, \frac{b_1}{a_2}, \frac{b_1}{a_1}\right)$ thus takes on negative values within the curve $P = 0$.

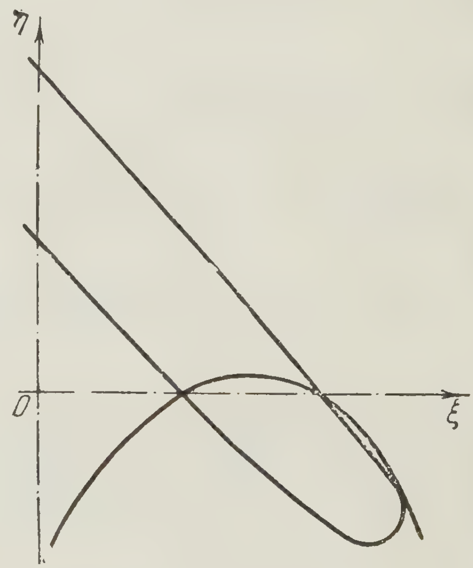

FIGURE 10

On the basis of the relative position of the curves $P = 0$ and $Q = 0$ with respect to each other, we can draw the following conclusion. For every value of the variable ϑ in the interval (b_2, b_1) we have $Q \geqslant 0$ if $P = 0$. Therefore, if the function $P(\vartheta)$ changes sign, the function $Q(\vartheta)$ will still be positive. From this the following results: if the Rayleigh function $R(\vartheta)$ is negative at the point $\vartheta = b_2$, then the change in its argument will be $-\pi$ as ϑ changes from ∞ to b_2. But this is the result which had to be proved.

Now it is easy to prove that the equation $R(\vartheta) = 0$ has no root in the interval $0 < \vartheta < b_2$ on the first sheet of the Riemann surface.

Indeed, suppose the Rayleigh function is negative at the point $\vartheta = b_2$, i. e., suppose $R(b_2) < 0$. If the function $R(\vartheta)$ should have a zero [root?] in $(0, b_2)$, then there would occur at least two changes in sign of $R(\vartheta)$ in $(0, b_2)$, for the function $R(\vartheta)$ is of the same sign (negative) near $\vartheta = 0$ and at the point $\vartheta = b_2$ (figs. 11 and 12). Therefore, the change in the argument of $R(\vartheta)$ must be equal to -2π [as ϑ varies] over the interval $(0, b_2)$ [When ϑ passes] around the point $\vartheta = 0$, the argument of $R(\vartheta)$ changes by -4π. In the interval $(b_2, \infty$ and $(-\infty, -b_2)$ the change in the argument of $R(\vartheta)$ is equal to -4π. Along the semicircle the increase in the argument of $R(\vartheta)$ is $+6\pi$. The total change in the argument of the function $R(\vartheta)$ along the closed contour is equal to

$$-\pi - 2\pi - 4\pi - 2\pi - \pi + 6\pi = -4\pi < 0.$$

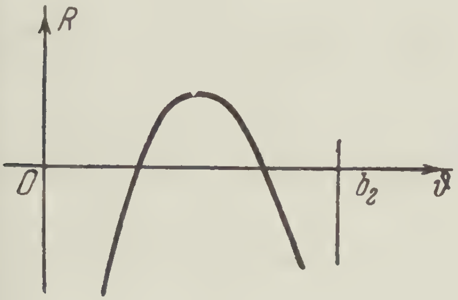

FIGURE 11

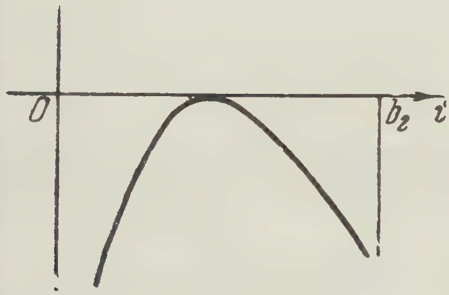

FIGURE 12

But this is impossible. Thus, there exists no root of $R(\vartheta)$ in the intervals $(0, b_2)$ and $(0, -b_2)$

Now we prove the absence of roots of the equation $R(\vartheta) = 0$ in the lower half-plane of ϑ. Because the equation $R(\vartheta) = 0$ has no root in the interval $(0, b_2)$ the total change in the argument of the function $R(\vartheta)$ along the closed contour will be equal to zero:

$$-\pi - 4\pi - \pi + 6\pi = 0,$$

This means that there are no zeros [roots?] of $R(\vartheta)$ in the lower half-plane.

We see that the condition $R(b_2) < 0$ guarantees the absence of roots [of $R(\vartheta) = 0$] of the first sheet of the Riemann surface.

Summarizing the preceding investigation, now we can formulate the necessary and sufficient conditions for the existence of a root of the Rayleigh equation. This condition is expressed by the inequality

$$R(b_2) \geqslant 0. \tag{106'}$$

where the velocity distribution is

$$b_2 < b_1 < a_1 < a_2 .$$

Investigation of Rayleigh's Equation for the Velocity Distributions $b_2 < b_1 < a_2 < a_1$, $b_2 < a_2 < b_1 < a_1$, $b_1 < b_2 < a_1 < a_2$, $b_1 < b_2 < a_2 < a_1$ и $b_1 < a_1 < b_2 < a_2$

First we examine the roots of the Rayleigh equation for the velocity distribution $b_2 < b_1 < a_2 < a_1$.

In the interval (a_1, ∞), on the first sheet of its Riemann surface, the Rayleigh function has the form:

$$\begin{aligned} R(\vartheta) = &\left[\vartheta^2\left(\frac{1}{b_1^2} - \frac{\delta}{b_2^2}\right) - 2(1-\sigma)\right]^2 + \\ &+ \frac{\sigma\vartheta^4}{b_1^2 b_2^2}\left(\sqrt{\frac{\vartheta^2}{a_1^2} - 1}\sqrt{\frac{\vartheta^2}{b_2^2} - 1} + \sqrt{\frac{\vartheta^2}{a_2^2} - 1}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\right) + \\ &+ \sqrt{\frac{\vartheta^2}{a_1^2} - 1}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\left(\frac{\sigma\vartheta^2}{b_2^2} + 2(1-\sigma)\right)^2 + \\ &+ \sqrt{\frac{\vartheta^2}{a_2^2} - 1}\sqrt{\frac{\vartheta^2}{b_2^2} - 1}\left(\frac{\vartheta^2}{b_1^2} - 2(1-\sigma)\right)^2 + \\ &+ 4(1-\sigma)^2\sqrt{\frac{\vartheta^2}{a_1^2} - 1}\sqrt{\frac{\vartheta^2}{b_1^2} - 1}\sqrt{\frac{\vartheta^2}{a_2^2} - 1}\sqrt{\frac{\vartheta^2}{b_2^2} - 1} . \end{aligned} \tag{107}$$

A change of the variable ϑ in the interval (a_2, a_1) will transform the Rayleigh function into the form

$$R(\vartheta)=\left[\vartheta^2\left(\frac{1}{b_1^2}-\frac{\sigma}{b_2^2}\right)-2(1-\sigma)\right]^2+\frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{\frac{\vartheta^2}{a_2^2}-1}\sqrt{\frac{\vartheta^2}{b_1^2}-1}+$$
$$+\sqrt{\frac{\vartheta^2}{a_2^2}-1}\sqrt{\frac{\vartheta^2}{b_2^2}-1}\left(\frac{\vartheta^2}{b_1^2}-2(1-\sigma)\right)^2-$$
$$-i\left[\frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_2^2}-1}+\right.$$
$$+\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\left(\frac{\sigma\vartheta^2}{b_2^2}+2(1-\sigma)\right)^2+$$
$$\left.+4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\sqrt{\frac{\vartheta^2}{a_2^2}-1}\sqrt{\frac{\vartheta^2}{b_2^2}-1}\right]. \qquad (108)$$

The argument $R(\vartheta)$ will vary over the interval $(0, -\frac{\pi}{2})$ as ϑ changes from $\vartheta = a_1$ to $\vartheta = a_2$

If ϑ lies within the interval (b_1, a_2) $R(\vartheta)$ will have the form:

$$R(\vartheta)=\left[\vartheta^2\left(\frac{1}{b_1^2}-\sigma\right)-2(1-\sigma)\right]^2-$$
$$-4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\sqrt{\frac{\vartheta^2}{b_2^2}-1}-$$
$$-i\left[\frac{\sigma\vartheta^4}{b_1^2 b_2^2}\left(\sqrt{\frac{\vartheta^2}{b_1^2}-1}\sqrt{1-\frac{\vartheta^2}{a_2^2}}+\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_2^2}-1}\right)+\right.$$
$$+\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{\frac{\vartheta^2}{b_1^2}-1}\left(\frac{\sigma\vartheta^2}{b_2^2}+2(1-\sigma)\right)^2+$$
$$\left.+\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{\frac{\vartheta^2}{b_2^2}-1}\left(\frac{\vartheta^2}{b_1^2}-2(1-\sigma)\right)^2\right]. \qquad (109)$$

Argument $R(\vartheta)$ changes within the interval $(0, -\pi)$.

At the point $\vartheta = b_1$ we have

$$R(b_1)=\left[1-\frac{\sigma b_1^2}{b_2^2}-2(1-\sigma)\right]^2-i\left[\frac{\sigma b_1^2}{b_2^2}\sqrt{1-\frac{b_1^2}{a_1^2}}\sqrt{\frac{b_1^2}{b_2^2}-1}+\right.$$
$$\left.+\sqrt{1-\frac{b_1^2}{a_1^2}}\sqrt{\frac{b_1^2}{b_2^2}-1}\left(1-2(1-\sigma)\right)^2\right.$$

Argument $R(b_1)$ lies within the interval $\left(0, -\frac{\pi}{2}\right)$.

For the interval $b_2 < \vartheta < b_1$ $R(\vartheta)$ has the following form:

$$R(\vartheta) = \left[\vartheta^2\left(\frac{1}{b_1^2} - \frac{\sigma}{b_2^2}\right) - 2(1-\sigma)\right]^2 - \frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}} -$$

$$-\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\sigma\vartheta^2}{b_1^2}}\left(\frac{\sigma\vartheta^2}{b_2^2} + 2(1-\sigma)\right)^2 -$$

$$-i\left[-4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_1^2}} +\right.$$

$$\left.+\frac{\sigma\vartheta^4}{b_2^1 b_2^2}\sqrt{1-\frac{\vartheta^2}{a_1^2}} + \sqrt{1-\frac{\vartheta^2}{a_2^2}}\left(\frac{\vartheta^2}{b_1^2} - 2(1-\sigma)\right)^2\right]\sqrt{\frac{\vartheta^2}{b_2^2} - 1}. \quad (110)$$

This obtained expression of the Rayleigh function is familiar to us. Therefore, we can assert that the criterion for the existence of a root of the Rayleigh equation remains unchanged, namely: for the existence of a Rayleigh wave it is necessary and sufficient that $R(b_2) \geqslant 0$.

Let us next consider the case of the velocity distribution when $b_2 < a_2 < b_1 < a_1$. The investigation of the existence of Rayleigh boundary waves for this case is analogous to the previous case; for, at the point $\vartheta = a_2$, $R(a_2)$ has a negative imaginary part:

$$R(a_2) = \left[c_2^2\left(\frac{1}{b_1^2} - \frac{\sigma}{b_2^2}\right) - 2(1-\sigma)\right]^2 -$$

$$-\sqrt{1-\frac{a_2{}^2}{a_1^2}}\sqrt{1-\frac{a_2{}^2}{b_1^2}}\left(\frac{\sigma a_2{}^2}{b_2^2} + 2(1-\sigma)\right)^2 -$$

$$-i\frac{\sigma a_2{}^4}{b_1^2 b_2^2}\sqrt{1-\frac{a_2{}^2}{a_1^2}}\sqrt{1-\frac{a_2{}^2}{b_2^2} - 1}.$$

The argument $R(a_2)$ can lie in the interval $(0, -\pi)$. This fact permits us to carry out the investigation of the roots of the Rayleigh equation [function] in a manner analogous to the one used above. Here it will turn out that the established criterion for the existence of a Rayleigh boundary wave remains unchanged.

Now we investigate the roots of the Rayleigh equation for the velocity distribution when $b_1 < b_2 < a_1 < a_2$, $b_1 < b_2 < a_2 < a_1$, and $b_1 < a_1 < b_2 < a_2$. The study of these cases is reduced to the previously considered types of velocity distributions.

Indeed, for the study of the question of the location of the roots of the Rayleigh equation it is necessary to examine the behavior of the Rayleigh function over the interval (b_1, b_2) in which this function $[R(\vartheta)]$ has the following form:

$$R(\vartheta) = \left[\vartheta^2\left(\frac{}{b_1^2} - \frac{\sigma}{b_1^2}\right) - 2(1-\sigma)\right]^2 - \sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{b_2^2}}\ \frac{\sigma\vartheta^4}{b_1^2 b_2^2} -$$

$$-\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_2^2}}\left[\frac{\vartheta^2}{b_1^2} - 2(1-\sigma)\right]^2 -$$

$$-i\left[-4(1-\sigma)^2\sqrt{1-\frac{\vartheta^2}{a_1^2}}\sqrt{1-\frac{\vartheta^2}{a_2^2}}\sqrt{1-\frac{\vartheta^2}{b_2^2}} +\right.$$

$$\left.+\frac{\sigma\vartheta^4}{b_1^2 b_2^2}\sqrt{1-\frac{\vartheta^2}{a_2^2}} + \sqrt{1-\frac{\vartheta^2}{a_1^2}}\left(\frac{\sigma\vartheta^2}{b_2^2} + 2(1-\sigma)\right)\right]\sqrt{\frac{\vartheta^2}{b_1^2} - 1}. \quad (111)$$

We see that here, in distinction from the previous cases, change of sign of the real imaginary parts of the Rayleigh function can occur when $\sigma > 1$.

The investigation of this case is now of fundamental importance. We can apply the method which

was used in the study of the function P and Q, and thus determine the criterion for the existence of a Rayleigh boundary wave. However, it is simpler to proceed in the following manner. We make the substitutions in the expression for $R(\vartheta)$

$$\frac{\vartheta^2}{b_2^2} = \zeta^2, \quad \sigma = \frac{1}{\sigma_1}, \quad \sigma = \frac{1}{\delta_1}, \quad \text{где} \quad \sigma_1 \frac{b_2^2}{b_1^2} = \delta_1. \tag{112}$$

These substitutions are equivalent to a transformation of the elastic media. On the basis of formula (111) we obtain:

$$\begin{aligned} R(\zeta_1) = \frac{1}{\sigma^2} \Big\{ & [\zeta^2 \sigma_1 - \zeta_1^2 - 2(\sigma_1 - 1)]^2 - \\ & - \sqrt{1 - \frac{\zeta^2 b_2^2}{a_2^2}} \sqrt{1 - \zeta^2} \, (\zeta^2 \sigma_1 - 2(\sigma_1 - 1))^2 - \\ & - \zeta^4 \delta_1 \sqrt{1 - \frac{\zeta^2 b_2^2}{a_1^2}} \sqrt{1 - \zeta^2} \Big\} - \\ - i \Big\{ & -4(\sigma_1 - 1)^2 \sqrt{1 - \frac{\zeta^2 b_2^2}{a_1^2}} \sqrt{1 - \frac{\zeta^2 b_2^2}{a_2^2}} \sqrt{1 - \zeta^2} + \\ & + \zeta^4 \delta_1 \sqrt{1 - \frac{\zeta^2 b_2^2}{a_2^2}} + (\zeta^2 + 2(\sigma_1 - 1))^2 \Big\} \sqrt{\frac{\zeta^2 b_2^2}{b_1^2} - 1}\,. \end{aligned} \tag{113}$$

The nature of the real and imaginary parts of $R(\vartheta)$ [$R(\zeta)$?] is the same as it was in the case of the previously considered velocity distributions. Therefore, it is easy to investigate this function. The necessary and sufficient condition for the existence of a Rayleigh boundary wave is given by the inequality $R(b_1) \geqslant 0$.

Criterion for the Existence of a boundary Rayleigh Wave. (Geometrical Interpretation)

Investigation of the Rayleigh function for arbitrary elastic media adjacent to each other along a plane, leads us to the following result: for the existence of a Rayleigh boundary wave it is necessary and sufficient that the Rayleigh function $R(\vartheta)$ be positive at the smallest value of the velocities in the elastic media under consideration. The Rayleigh boundary wave, if it exists, will have a propagation velocity less than that of longitudinal or transverse waves. Because of the fact that the root of the Rayleigh equation is a real number, we can assert that the existence of the Rayleigh [boundary] wave, and its velocity, depend only on the physical constants of the considered media and not on the length or frequency of the wave. Se shall give a geometrical interpretation of the criterion for the existence of a Rayleigh boundary wave. Let us assume that $b_2 < b_1 < a_1 < a_2$. For the purpose of finding the numerical values of the elastic constants and the densities for which Rayleigh boundary waves can exist, we construct the parabola $P(b_2) = 0$ (fig. 13) on the ξ, η plane for positive values η, , where:

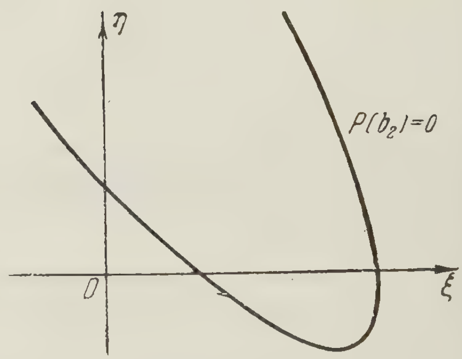

FIGURE 13

$$\xi = 2\left(1 - \frac{\mu_2}{\mu_1}\right), \quad \eta = \delta = \frac{\rho_2}{\rho_1},$$

$$\begin{aligned} P(b_2) = & \left[\frac{b_2^2}{b_1^2}\eta + \xi\right]^2 - 2\frac{b_2^2}{b_1^2}\left[\frac{b_2^2}{b_1^2}\eta + \xi\right] \cdot \\ & + \frac{b_2^4}{b_1^4} - \sqrt{1 - \frac{b_2^2}{a_2^2}} \sqrt{1 - \frac{b_2^2}{b_1^2}\frac{b_2^4}{b_1^4}\eta} - \\ & - \sqrt{1 - \frac{b_2^2}{a_1^2}} \sqrt{1 - \frac{b_2^2}{b_1^2}} \left(\frac{b_2^2}{b_1^2}\eta + \xi\right) = 0. \end{aligned} \tag{114}$$

Those points of the place for which $\eta = \frac{b_1^2}{b_2^2}\left(1 - \frac{\xi}{2}\right)$, and which lie within the

parabola $P(b_2) = 0$, determine a class of elastic media; contact with them will not produce Rayleigh waves. The points of the (ξ, η) plane that lie outside the parabola $P = 0$, determine elastic media; contact with them will give rise to Rayleigh boundary waves.

In the general case, the dimensions of the parabola $P = 0$ are determined by the elastic constants.

General Formulas for the Reflection and Refraction of Plane Elastic Waves

Elastic disturbances, propagated in various elastic media that are adjacent to each other along a plane, can be treated as a set of longitudinal and transverse waves that are reflected and refracted with various real or complex angles. If the elastic constants satisfy the inequality $R(b_2) > 0$, when $b_2 < b_1$; or $R(b_1) > 0$ when $b_1 < b_2$, then Rayleigh boundary waves will occur in addition to these other oscillations. Our problem is to obtain general formulas for the reflection and refraction of [plane] elastic waves; these formulas are to contain all of the oscillations that we have considered as special cases.

The solutions of the wave equations (1) and (2) can be given, according to D'Alembert, in the form:

$$\varphi_1 = \Phi_1\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right) + \Phi_2\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right),$$

$$\psi_1 = \psi_1\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right) + \psi_2\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right). \quad (115)$$

$$\varphi_2 = \Phi_3\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\right) + \Phi_4\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\right), \quad (116)$$

$$\psi_2 = \psi_3\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right) + \psi_4\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right). \quad (117)$$

Let $|c| = \left|\frac{p}{\alpha}\right| > a_1 > b_1 > a_2 > b_2.$

The substitution of the solutions (116, 117) into the boundary conditions (4) leads us to the equations:

$$\begin{gathered}\left(\frac{p^2}{b_1^2} - 2\alpha^2\right)\Phi_2 + 2\alpha\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\,\psi_2 - \sigma\left(\frac{p^2}{b_2^2} - 2\alpha^2\right)\Phi_4 + \\ + 2\sigma\alpha\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\,\psi_4 = -\left(\frac{p^2}{b_1^2} - 2\alpha^2\right)\Phi_1 + 2\alpha\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\,\psi_1 + \\ + \sigma\left(\frac{p^2}{b_2^2} - 2\alpha^2\right)\Phi_3 + 2\sigma\alpha\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\,\psi_3; \quad (118)\end{gathered}$$

$$\begin{gathered}-2\alpha\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\,\Phi_2 + \left(\frac{p^2}{b_1^2} - 2\alpha^2\right)\psi_4 - 2\sigma\alpha\sqrt{\frac{p^2}{a_2^2} - \sigma^2}\,\Phi_4 - \\ - \sigma\left(\frac{p^2}{b_2^2} - 2\alpha^2\right)\psi_4 = -2\alpha\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\,\Phi_1 + \left(\frac{p^2}{b_1^2} - 2\alpha^2\right)\psi_1 - \\ - 2\sigma\alpha\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\,\Phi_3 + \sigma\left(\frac{p^2}{b_2^2} - 2\alpha^2\right)\psi_3;\end{gathered}$$

$$\alpha\Phi_2 - \sqrt{\frac{p^2}{b_1^2} - \alpha^2}\,\psi_2 - \alpha\Phi_4 - \sqrt{\frac{p^2}{b_2^2} - \alpha^2}\,\psi_4 = -\alpha\Phi_1 - \sqrt{\frac{p^2}{b_1^2} - \alpha^2}\,\psi_1 + \alpha\Phi_3 - \sqrt{\frac{p^2}{b_2^2} - \alpha^2}\,\psi_3; \tag{119}$$

$$-\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\,\Phi_2 - \alpha\psi_2 - \sqrt{\frac{p^2}{a_2^2} - \alpha^2}\,\Phi_4 + \alpha\psi_4 = -\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\,\Phi_1 + \alpha\psi_1 - \sqrt{\frac{p^2}{a_2^2} - \alpha^2}\,\Phi_3 - \alpha\psi_3.$$

We express the potentials Φ_1, ψ_1, Φ_3, and ψ_3 (for the sake of simplicity) in the following form:

$$\Phi_1 = R(p,\alpha) f_1,\ \psi_1 = R(p,\alpha) f_2,\ \Phi_3 = R(p,\alpha) f_3.\ \psi_3 = R(p,\alpha) f_4,$$

where f_j $(j = 1, 2, 3, 4)$ is an arbitrary function of a real variable and $R(p,\alpha)$ is the Rayleigh function. Solution of the systems (118, 119) for Φ_2, ψ_2, Φ_4 and ψ_4 we write in the form:

$$\varphi_1 = R(p,\alpha) f_1\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right) + \sum_{j=1}^{4} l_j(p,\alpha) f_j\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_1^2} - \alpha^2}\right),$$

$$\psi_1 = R(p,\alpha) f_2\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right) + \sum_{j=1}^{4} m_j(p,\alpha) f_j\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_1^2} - \alpha^2}\right), \tag{120}$$

$$\varphi_2 = R(p,\alpha) f_3\left(pt + \alpha x - y\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\right) + \sum_{j=1}^{4} n_j(p,\alpha) f_j\left(pt + \alpha x + y\sqrt{\frac{p^2}{a_2^2} - \alpha^2}\right),$$

$$\psi_2 = R(p,\alpha) f_4\left(pt + \alpha x - y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right) + \sum_{j=1}^{4} p_j(p,\alpha) f_j\left(pt + \alpha x + y\sqrt{\frac{p^2}{b_2^2} - \alpha^2}\right), \tag{121}$$

where the coefficients l_j, m_j, n_j p_j $(j = 1, 2, 3, 4)$ represent fourth order determinants obtained by solution of the system (118, 119).

The function f_j, can be considered as the real parts of certain functions S_j [?] that are regular in the upper complex half-plane [S_j]; i. e., $Re\, S_j = f_j$. It is easy to prove that the solution of the general oscillation problem can be written in the form

$$\left.\begin{aligned}
\varphi_1 &= Re\left\{R(p,\alpha)S_1\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\right)+\right.\\
&\left.+\sum_{j=1}^{4} l_j(p,\alpha)S_j\left(pt+\alpha x-y\sqrt{\frac{p^2}{a_1^2}-\alpha^2}\right)\right\},\\
\psi_1 &= Re\left\{R(p,\alpha)S_2\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)+\right.\\
&\left.+\sum_{j=1}^{4} m_j(p,\alpha)S_j\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_1^2}-\alpha^2}\right)\right\},\\
\varphi_2 &= Re\left\{R(p,\alpha)S_3\left(pt+\alpha x-y\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\right)+\right.\\
&\left.+\sum_{j=1}^{4} n_j(p,\alpha)S_j\left(pt+\alpha x+y\sqrt{\frac{p^2}{a_2^2}-\alpha^2}\right)\right\},\\
\psi_2 &= Re\left\{R(p,\alpha)S_4\left(pt+\alpha x-y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)+\right.\\
&\left.+\sum_{j=1}^{4} p_j(p,\alpha)S_j\left(pt+\alpha x+y\sqrt{\frac{p^2}{b_2^2}-\alpha^2}\right)\right\}.
\end{aligned}\right\} \quad (122)$$

Indeed, let 1) $|c| = \left|\frac{p}{\alpha}\right| > a_1$.

Then formula (122) represents the most simple oscillations which can be described as real-plane waves.

Let us consider the change in the phase velocity **c** in the interval

$$2)\ b_1 < |c| < a_1.$$

In this case the function S_1 has for its argument the expression

$$pt + \alpha x \pm iy\sqrt{\alpha^2 - \frac{p^2}{a_1^2}},$$

Therefore, on the basis of the hypotheses imposed on S_1, this function must be bounded on the entire complex plane; hence, by Liouville's theorem, the function S_1 has to be a constant. It is easily shown that the formula (122) satisfies the boundary conditions.

The potentials φ_2, ψ_2, ψ_1 represent uniform-(real-) plane waves; with regard to the longitudinal-wave potential φ_2, one can say that it represents a nonuniform-(complex-) plane wave.

In the interval 3) $a_2 < |c| < b_1$, the argument $pt + \alpha x \pm iy\sqrt{\alpha^2 - \frac{p^2}{b_1^2}}$ has the form of a complex quantity for S_2. Therefore, S_2 can be considered a constant, and can thus be excluded from formula (122).

Let us consider the values of c which satisfy the the inequality 4) $b_2 < |c| < a_2$. Here, it is necessary to treat S_1, S_2 and S_3 as constants which can be neglected in the dynamical problem. Substituting the remainder part of the expression in (122) into the boundary condition (4), one can easily establish that these conditions are satisfied.

Finally let us consider the value of **c** which satisfy the inequality 5) $|c| < b_2$. It is easily seen that out of all possible values satisfying inequality (5), the nontrivial solutions are given by two values $\pm\,\varepsilon$, and ε is a root of the Rayleigh equation $R(\vartheta) = 0$. If $R(\pm\,\varepsilon) = 0$ the coefficient of the first term in the formula (122) has the value zero. Because of the Rayleigh equation there exist relations among the

coefficients l_j, m_j, n_j, p_j, and the expressions for the potentials contain a linear combination of the functions S_j. Without loss of generality, one can assume that $S_2 = S_3 = S_4 = 0$, and we thus obtain formulas which represent a Rayleigh wave.

Thus, all possible solutions of the oscillation problem represent the set of all uniform-(real-) and nonuniform-(complex-) plane waves.

On the basis of the preceding investigation, one can apply the well known method of V. I. Smirnov and S. L. Sobolev for the solution of dynamical problems in the theory of elasticity for the case when several elastic media are in contact with each other along a plane, or in the case of layered media (interference case).

The next investigation will be devoted to the analysis of waves produced by arbitrary sources of disturbances and under arbitrary initial conditions.

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Notes on international scientific meetings

ON THE RESULTS OF THE SCIENTIFIC-TECHNICAL GEOPHYSICAL CONFERENCE[1] (U.S.S.R.)

by A. S. Shirokov[2]

• *prepared by the United States Joint Publications Research Service[3]* •

A scientific-technical geophysical conference organized by the Ministry of Geology and Conservation of Mineral Resources of the U. S. S. R., the State Scientific-Technical Committee of the Council of Ministers of the U. S. S. R., and the Scientific-Technical Mining Society was held in Moscow on 6-8 October 1959.

749 delegates from 154 industrial, scientific-research, experimental-design establishments, and other organizations of the U. S. S. R., Ministry of Geology and Conservation of Mineral Resources, the U. S. S. R.; State Planning Commission, Academies of Science of the U. S. S. R., and Union republics, Soviet Economic Councils (Sovnarkhozov), and from scientific-research and training institutes, participated in the work of the conference.

In an introductory speech, B. N. Erofeyev, Deputy Minister of Geology and Conservation of Mineral Resources of the U. S. S. R., noted that geophysical research methods (whose volume will be increased over 2 times during the Seven-Year Plan) play a major role in prospecting for mineral deposits. The conference must be devoted to the cause of raising further the level of geophysical work and its geological efficiency, and encouraging wide circles of geophysicists and geologists to fulfill the historical directives issued at the XXI Congress and at the June Plenum of the Central Committee of the Soviet Union Communist Party.

The Conference examined problems concerning the status, trends, methods, and perspectives in scientific-research development, experimental design, instrument building and applied-geophysical work carried out by various establishments and organizations in the Soviet Union.

More than 300 delegates participated in discussion of the 165 reports and communications presented at the conference, which closed by adopting a sweeping resolution.

V. V. Fedynsky, a Board member of the Ministry of Geology and Conservation of Mineral Resources of the U. S. S. R., and chief of the Geophysical Department, read an important basic paper entitled "The Basic Tasks and Prospective Development of Geophysical Operations Concerned With the Prospecting and Survey of Mineral Resources During the 1959-1965 period".[4] The speaker pointed out that the volume of operations performed by all known geophysical methods has been significantly increased during the current seven-year period and their distribution among the various regions and mineral resources has been improved. The ratio of geophysical survey methods in the general complex of geological prospecting operations will be increased from 15.4 percent in 1958 to 25 percent in 1959. A significant amount of geophysical work will be conducted in the eastern regions of the Soviet Union. The paper stressed the role played by scientific-research institutes in developing and improving procedures and techniques of individual geophysical survey methods and their combined application in all stages of geological survey work.

The paper read by A. S. Shirokov and V. V. Zhuravlev (Geophysics Department of the Ministry of Geology and Conservation of Mineral Resources U. S. S. R.), entitled "Status of Technical Equipment and Prospective Development of Geophysical Instrument Construction",[5] gave a detailed description of the present status and of basic methods for intensifying scientific-research and experimental-design work in the field of geophysical instrument construction.

The paper read by A. N. Tikhonov (Council on Methods of Prospecting Geophysics), and E. V. Karus (Institute of Earth Physics of the U. S. S. R. Academy of Sciences) entitled "Status and Research Trends in the Development of New and the Improvement of Present Methods of Prospecting Geophysics at the U. S. S. R. Academy of Sciences and at

[1] Translated from K itogam raboty nauchno-tekhnicheskoy geofizicheskoy konferentsii: Razvedka i okhrana nedr, Gosgeolizdat, no. 12, 1959, p.54-59.

[2] Ministry of Geology and Conservation of Mineral Resources, U.S.S.R. 3

[3] JPRS: 3087

[4] "Osnovnyye zadachi i perspektivy razvitiya geofizicheskikh rabot po poiskam i razvedki poleznykh iskopayemykh na 1959-1965 gg."

[5] "Sostoyaniye tekhnicheskoy vooruzhennosti i perspektivy razvitiya geofizicheskogo priborostroyeniya."

Academies of Sciences of Union Republics",[6] and the paper by M. K. Polshkov (All-union Scientific Research Institute of Geophysics) entitled "Status and Development Trends of Scientific-Research Work in Applied Geophysics in the U. S. S. R. ",[7] were concerned principally with description of a wide range of problems concerning theoretical developments and the creation of new and advanced geophysical survey methods (including aerial geophysical methods), the design of more efficient geophysical instruments and equipment used in all metgods of geophysical prospecting, and the mechanization and automation of geophysical operations.

T. N. Simonenko and T. N. Spizharskiy (All-Union Geological Scientific-Research Institute) read a paper entitled "The Use of Geophysical Data in Drawing a Tectonic Map of the U. S. S. R., on a Scale of 1:2,500,000",[8] showing the advantages of a geological map based on the use of geophysical data.

The report made by B. V. Kotlyarevsky (Geophysics Department) and L. A. Ryabinkin (MINKHIGP [Moscow Inst-Res Chem/Geophy Equip Constr] entitled "Status and Development Trends of Seismographic Geophysical Exploration, "[9]noted that over 60 percent of the funds allocated to geophysics in the current Seven-Year Plan will be spent on seismic-geophysical exploration work. Therefore, it is necessary in the first place to improve methods and techniques used in seismic-geophysical exploration and to introduce complex mechanization and automation of production into geophysical methods.

A large number of reports and communications were presented and discussed in the seven sections that worked simultaneously during the conference.

In the Structural -Geophysics Section 19 reports and communications were presented and discussed: these were concerned with regional geophysical surveys of folded substructure and of sedimentary cover in various regions of the U. S. S. R.; with study of the structure at depth of the earth's crust; and, with use of geophysical exploration methods in oil and gas prospecting.

As was noted in the papers read by V. I. Kulikov, M. V. Chervinskaya, L. I. Ivanov, and others, correct orientation of survey and prospecting work for oil, gas, and other types of mineral resources can be achieved in a number of regions with the aid of structural and geological diagrams obtained in regional geophysical surveys.

The section noted that new modifications of regional geophysical survey methods have been developed in recent years, such as the TT electric geophysical exploration method and the KMPV seismic-geophysical exploration method. As was noted in the reports of Yu. N. Godin and I. P. Kosminskaya, surveys of the earth's crust with the aid of deep seismic probings are practiced extensively; thus allowing one to trace the course of plutonic fractures, and, to study separate levels.

The problem concerning the use of combined geophysical methods including seismic- and gravimetric-prospecting, and radiometric methods for direct prospecting of oil and gas deposits in Bashkiria, Azerbaijan, and Turkmenia, was examined in papers read by F. A. Alekseyev and I. G. Medovsky.

At the same time, the section noted the inadequate development of regional surveys. There is still a lack of a sufficiently complete study of geological structure, based on geophysical data; especially with respect to regions of Siberia and the Far East. Accuracy of the cartographic drawing of substructure contours sometimes does not comply with the necessary requirements. The section recommended that work on deep seismic probing be continued in order to study the deep (plutonic) structure of the earth's crust in dry land areas, in regions of different geological structure, and in seas and oceans as well.

Regional geophysical surveys should be used widely in the systematic study of geological structure of depth of the territory of this country, to estimate the mineral potential of individual regions and to establish a scientific basis for subsequent survey and prospecting operations to conduct geological mapping of depth together with geological surveying and drilling operations, and to draw geological maps on a 1:200,000 scale. A recommendation was made to increase significantly the volume of scientific-research work aimed at improving combined regional geophysical surveys and the methods used for conducting such surveys under geological conditions; as well as work on the development of direct prospecting methods for locating oil and gas deposits; using seismographic and gravimetric prospecting techniques; and radiometric methods.

In the Mineral Geophysics Section, a total of

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[6] "Sostoyaniye i puti issledovany po sozdaniyu novykh i usovershenstvovaniyu syshchestvuyushchikh metodov razvedochnoy geofiziki v Akademii Nauk SSSR i v Akademii Nauk soyuznykh respublik."

[7] "Sostoyaniye i puti razvitiya nauchno-issledovatelskikh rabot v SSSR v oblasti prikladnoy geofiziki."

[8] "Ob ispolzovanii geofizicheskikh dannykh pri sostavlenii tektonicheskoy karty SSSR masshtaba 1:2,500,000.'

[9] "O sostoyanii i putyakh razvitiya seysmorazvedki."

23 reports and communications on various questions concerned with the prospecting of mineral deposits, were presented. These questions included the general status and development of geophysical operations during complex prospecting of mineral deposits; results of the application of geophysical methods in mapping belt- (closed-) ore regions; and a number of specific projects concerned with the development of equipment, methods, etc.

The great achievements of geophysicists in developing combined geological and geophysical prospecting methods for locating mineral deposits in Kazakhstan and Uzbekistan; the positive results of the application of geophysics in geological mapping work conducted in the Transbaykal region and in the prospecting of rich iron ore in the Kursk magnetic anomaly and in Western Siberia; and in studying the diamond fields of Yakutiya and copper deposits in the Urals; etc. were pointed out in the reports presented by A. D. Miller, A. P. Solovov and others.

Of great interest were reports describing the development of new mineral geophysical methods; such as radiography (A. D. Petrovsky); the "radiokip" method (A. D. Frolov); methods of induced polarization and underground gravimetry; as well as a report by A. G. Tarkhov, concerning certain additions to information theory in exploratory geophysics. The reports showed the significant expansion in the potential application of mineral geophysics.

At the same time, the section noted the very low degree of efficiency in geophysical operations during prospecting for nonferrous and rare-metal deposits, especially "blind" deposits and ore bodies located at great depths. The use of geophysical methods in indirect prospecting for mineral deposits; in tracing ore-indicating symptoms; and, in studying the structure of mineral deposit regions, is not given the proper amount of consideration. In general, the development and improvement of combined regional geophysical surveys during prospecting of useful mineral deposits, are still lagging behind industrial requirements. The section suggests that mining enterprises should concentrate their efforts on conducting regional complex geophysical operations prior to prospecting work. In locating and studying mineral deposit regions, and in order to study their structure and factors that will facilitate mineral prospecting work, it was considered expedient to carry out large-scale complex geophysical surveys on a 1:50,000 and 1:10,000 scale in conjunction with geochemical methods and prospecting-surveying operations together with drilling and mining operations.

Work involving generalization of available geophysical materials, aimed at clarification of their geological interpretation and selection of a rational combination of surveying methods as well as detailed study of the physical properties of rocks and minerals, will be of great help in raising the efficiency of mineral geophysics. The section suggests that scientific-research organizations engage in more intensive work aimed at development and introduction of new geophysical methods and improvement of presently used methods for prospecting and exploring mineral deposits; and, that they should develop rational complex methods applicable to conditions prevalent in individual mineral regions as well. Particular attention must be given to further elaboration of theory on presently used and newly developed mineral geophysical methods.

The Seismic Exploration Section heard 26 reports and communications concerned with the theory and general problems in the field of seismic-geophysical exploration, development of new seismic-exploration equipment, problems on method, and the results of seismic exploration in oil- and gas-bearing regions.

Of exceptionally great interest were the reports presented by I. S. Berzon, concerning the study of dynamic characteristics of seismic waves in real media; by N. N. Puzyrev, on the registration of transverse waves; and, by A. M. Epinatyeva on multiple reflected waves. Among the reports having an important practical significance, one might mention in the first place the papers read by A. O. Slutskovsky, A. N. Fedorenko, and others, concerning design of new models of seismic stations; by S. Ya. Rappoport, and others, on marine seismic exploration; by A. K. Shmelev, on river seismic exploration; by V. D. Zavyalov, on a method of spatial seismic probing; by L. D. Rayker, on the development of a plane-front method; and, by E. V. Karus and others, on the study of physical and mechanical properties of rocks by a pulse supersonic core-sampling method; etc.

The section noted that seismic exploration is the most accurate geophysical method in use at the present time. In a number of regions in the Caspian lowland area and other regions, seismic exploration is the basic method used in preliminary deep-drilling operations performed in promising oil- and gas-bearing structures. Mining enterprises have scored substantial achievements in the development of seismographic exploration methods. The plane-front method and the spatial seismic probing method have been developed and are being used successfully by the Ukrainian Geophysical Trust; and, a river seismic-exploration method has been developed by the Tyumen Oil Exploration Trust.

Within the past few years, the following methods have been developed and used in production correlation method of refracted waves; the seismic deep-probing method; the controlled guided-reception method; frequency-modification seis-

mic-exploration methods (VChS- or high-frequency seismic exploration, and NChS- or low-frequency seismic exploration) and marine-seismic exploration. The following new seismic equipment has been designed and is now used: a seismographic set with magnetic recording (SSM-57, PPMZ-2), a portable seismographic set (SS-24 P), seismographic sets (RNP), marine seismographic sets (MSS-58), piezoseismographic spits [sic] (or scythes ?), and various types of seismographic receivers.

Considerable progress has been made at scientific-research institutes in developing methods for interpreting seismic data. However, seismic exploration is not capable of solving geological problems in all regions. The geological efficiency of seismic exploration methods is particularly low in the prospecting of sloping platform-type structures, in studying complex structures, and in prospecting for mineral deposits. As a rule, anticline sections of tectonically disrupted upheavals are not exposed by seismic exploration; in many regions the exposure depth of the pit is not sufficiently great. The cost of seismic exploratory operations is extremely high in a number of regions, especially in Siberia and in the Far East.

The section outlined the following principal work trends: development and introduction of operational methods utilizing reproducible recordings and devices for the automatic data processing; development of a theory for grouping receivers and oscillation sources; development of methods allowing utilization of new types of waves (transverse, exchange, and diffracted waves) for obtaining additional data on a geological profile, and use of dynamic characteristics of seismic waves during interpretation. Particular attention was given to providing industrial enterprise with modern seismic instruments, equipment, and all auxiliary materials and means of transport.

The Electric Geophysical Exploration Section heard 22 reports and communications dealing with development of methods and equipment based on the procedures and results of application of electric geophysical exploration to solving geological problems. A large number of interesting reports and communications were presented by scientific-research institutes (A. N. Tikhonov, N. M. Shuval-Sergeyev, L. Ya. Mizyuk, L. L. Vanyan, V. A. Komarov, N. P. Silin, S. M. Sheyman and others).

The section noted a considerable increase, during the past few years in the volume of electric geophysical exploration work for solving problems of structural geology, and in prospecting and surveying mineral deposits during hydrogeological and geological engineering surveys, and during geological mapping operations; the range of geological problems solved with the aid of this method also has expanded. Procedures and equipment used in a number of new electric geophysical-exploration methods have been developed and introduced; or, are in a final development stage as a result of work done at scientific-research institutes, industrial organizations, design bureaus, and geophysical-instrument-construction plants. Such new methods include dipole electric probing, marine-electric exploration, electric-frequency probing methods, magnetotelluric profiling, aerialelectric-exploration methods, as well as methods utilizing telluric currents, establishment of an induced potential field, and radiowave translucence and "radiokip" methods. Standard electric exploration sets ERS-23, ERS-16.5 and EPL-57, and electric compensators ESK, KSRM, and EAK have been designed and are now in use.

The section noted that, at the present time, there is a definite possibility for making greater use of electric exploration methods in solving structural geology problems by using methods less costly than seismic exploration; i. e. telluric current, magnetotelluric profiling, and field formation.

The use of aerial electric exploration methods together with inductive ground methods in mineral geophysics, will result in more rapid study of mineral regions and in more definite determination of mineral and nonmineral anomalies.

The section recommends an increase in the volume of electric exploration work on prospecting-surveying and exploration operations; and, suggests intensification of scientific-research and design work aimed at developing and adopting new electric exploration methods and equipment, and improving presently used methods and instruments. Such work should involve, primarily, completion of the development of the theory, methods, and equipment used in electromagnetic frequency probing as well as of procedures and instruments used in the field-formation method, in order to solve structural-geological problems connected with mineral deposits. The conference recommends that work on the theory of interpreting data obtained in electric exploration surveys be accelerated considerably, and , that computer devices be used.

Gravimetric Exploration Section heard 21 reports devoted to various problems concerning methods and techniques used in gravimetric operations, and, to prospects of their further development. Great interest was expressed in the report presented by L. V. Petrov describing the tasks confronting gravimetric exploration in the current Seven-Year Plan, and, that by K. E. Veselov, S. A. Poddubny, B. A. Andreyev, and others, describing the development status of new equipment and methods used in gravimetric-exploration-data interpretation.

The section noted that considerable progress has been made in the field of gravimetric

exploration in recent years. The volume of gravimetric exploration operations has increased; technical equipment has been improved; the range of applied methods has been expanded; and, geological efficiency of these methods has been raised. At the present time, gravimetric exploration, combined with magnetic exploration in support of seismic-traverse routes and drilling data, is one of the principal methods used in studying geological structure of the earth's crust at depth, and in geological mapping. In many regions, gravimetric exploration is used in prospecting for oil, gas and mineral deposits. Substantial methodical achievements have been made in marine and underground gravimetric exploration, in the study of rock densities, and in developing topographic and geodetic measures, making it possible to conduct gravitational surveys. Great progress has been made in gravimetric-exploration-data interpretation in methods used for classifying gravitational anomalies, aligning anomalies, etc.

The section noted that effectiveness of gravimetric exploration remains inadequate in many cases and does not stand up to industrial requirements. The resolving power of gravimetric exploration in prospecting for mineral deposits and enclosing rocks was found to be inadequate in a number of cases.

The section outlined the following basic problems: the conduct of regional surveys combined with other geophysical methods and drilling; 2-milligal and [1-] milligal surveys for geologic mapping purposes; application of surveys with gravimeters, variometers, and gradient meters for ore-deposit prospecting amd exploration; and, in particular, for deposits located at great depths, the accelerated development and serial manufacture of high-precision gravimeters of the GAK-6m type and gravimeter-altimeters of the GVP-1 type. It was deemed necessary to improve radiogeodetic instruments (both air and ground modifications); and, to design improved computing devices for gravimetric-exploration-data processing and interpretation.

The Magnetic Exploration Section heard 24 reports and communications describing problems concerned with the theory and practice of magnetic-exploration-data interpretation, with geological results of aerial-magnetic surveys, and with methods of compiling magnetic charts and instrument building. Of great interest were theoretical papers describing the theory of the geophysical work performed by I. G. Klushin and S. V. Shalayev, in which problems concerning methods of interpreting magnetic anomalies were examined, as well as the paper read by F. N. Efimov concerning the development of methods for conducting a fractional-mineralogical analysis of rocks.

In a summary report, presented by V. E. Nikitsky, V. I. Fedyuk and T. M. Simonenko, and in other reports as well, it was noted that considerable progress was achieved in recent years in the field of magnetic exploration during regional surveys; geological mapping and prospecting for mineral resources; and, in the study of magnetic properties of rocks. In addition, the range of problems which are being solved with the aid of magnetic exploration, used at the present time in combination with other geophysical methods, has been expanded significantly.

In recent years, aeromagnetic surveys have found a particularly wide field of application during studies of the (plutonic) geological structure at depth of large areas, and in geological mapping operations. The geological effectiveness of aeromagnetic surveys was demonstrated clearly [by the example] of work done in the western Siberian lowland, in Uzbekistan, in the Kurile and Kamchatka island region, and in the Antarctic region.

In using a number of new modifications of ground magnetic exploration, favorable results were obtained by the following methods: the micromagnetic survey method (used on kimberlite pipes of Yakutiya); measurements of gradients in magnetic-field components (Krivoy Rog); and, use of variations in magnetic-field elements for classification of magnetic anomalies (Eastern Sayan region).

The following new instruments and equipment used in magnetic exploration will soon be manufactured: a high-percision aeromagnetometer A M-13; the ASG-45 set; the magnetovariational set SMV-2; the M-14 and M-16 ground quartz magnetometers; and, the portable magnetometer M-17. The design of instruments for automatic processing of magnetic-survey data is in progress. Methods involving mathematical interpretation of magnetic-field anomalies, have been further developed.

The section noted the presence of serious shortcomings in the field of magnetic exploration, in spite of a certain amount of progress achieved in this area. Aeromagnetic and ground operations have not been supplied yet with adequate technical equipment and facilities. Methods for conducting large-scale surveys, etc. have not been developed to a sufficient extent.

The section recommended continued development and industrial application of new methodical procedures for conducting ground and air surveys; these would provide the necessary high-accuracy standards required in such operations. The section also recommended that work on the creation of an All-Union supporting magnetic network, be conducted within the next 2 to 3 years. The design of equipment

should include design of optical and mechanical instruments provided with compensation pickups of the second-harmonic type; including the development of guidance systems, and, of methods for excluding interference signals and allowing the continuous registration of signals.

The section also passed a resolution calling for an intensification of theoretical and experimental work aimed at developing and introducing radiogeodetic means for tying in the course of an aeromagnetic survey; and, development of new methods for the processing and quantitative interpretation of magnetic anomalies, involving the use of computing devices. In addition, development work on problems concerned with the theory and practice of magnetic exploration, as applied to study of ore-field structures and exploration of ore-field structures and exploration of ore deposits, should be expanded substantially; and, the volume of large-scale aeromagnetic surveys (1:50, 000 and 1:25, 000) should be increased sharply.

The Geophysical Bore-Hole Survey Section heard 23 reports and communications dealing with methods and techniques used in core-sampling (logging) operations; with development prospects of scientific-research and experimental design work; and, with design and output of new improved instruments and equipment.

The section noted that in recent years as a result of improvement of those presently available, and development of new types of improved instruments and equipment, productivity and geological efficiency of logging operations have been increased considerably; and, that the costs of such operations have been reduced significantly.

The combined-study methods used at the present time in oil and gas wells, make it possible to isolate samplers as well as to estimate their oil and gas content in most regions. In a number of regions, the collecting properties of layers are determined in order to calculate available oil and gas reserves.

In coal deposits, bore hole pits are correlated by means of core-sampling operations; these also permit one to establish the presence of coal layers and to determine their thickness and structure. As a result of the extensive use of the potentialities presented by core-sampling techniques, coreless and partially coreless mine-pit-drilling methods are being introduced in a number of coal-bearing areas.

Geophysical studies are conducted on a wider scale in mineral bore holes; where, layers are broken down according to geological differences, and ore bodies (iron ores, sulfides, and others) are separated.

In this section, reports and communications presented by V. N. Dakhnov, I. I. Feldman, K. N. Yakubson, and others, described the development of new methods for studying bore holes; i. e. selective gamma-gamma core sampling which allows quantitative determination of heavy elements present in ores (lead, tungsten, molybdenum, mercury); activation analysis for estimating copper, aluminum, and manganese content; photoneutron core sampling used in prospecting and reviewing rare-element deposits; investigations of oil, gas, and ore drillings, based on data on induced-polarization potentials.

A number of reports such as those presented by S. M. Akselrod, V. N. Ponomarev, and others described the results of work performed on the design of new equipment; e. g. design of bore-hole neutron generators; magnetic and induction core-sampling equipment; a new type of bore-hole cement meter, which makes it possible to check the distribution of cement behind the column following cementing without the use of a radioactive source; a lateral drilling core-lifter; and, a number of other geophysical instruments.

At the same time, the section noted a number of serious short-comings in core-sampling operations. The efficiency of geophysical studies performed in bore holes is still not always sufficiently high; and, a number of problems still remain unsolved up to the present time. There are no reliable methods for isolating collectors in carbonate layers and fissured rocks methods of determining the collecting properties of oil and gas layers are not being introduced to a sufficient extent into operational practice, and basic theoretical and experimental work is lagging behind.

Reliable methods for obtaining qualitative coal characteristics derived from results of geophysical studies are not available; this fact prevents the development of highly productive and economic coreless drilling methods.

Core-sampling operations are performed to a much smaller extent in ore deposits than in oil and coal deposits. Methods used in determining the presence of minerals and their percentage composition, are being developed at a slow rate.

The development of new promising core sampling methods, such as lateral, acoustic, ultrasonic, and other types of core sampling, also is proceeding at a slow rate. So far, no combined core-sampling sets and drilling instruments capable of simultaneously measuring several (4 or 5) parameters during a single drilling pass, have been designed.

The volume of scientific-research and design work aimed at developing new geophysical research methods and designing new equipment,

is completely inadequate.

The section recommended expansion of scientific-research work in the field of geophysical studies of drillings in order to increase the effectiveness of separating and estimating oil and gas collectors in carbonate deposits, and of fissured collectors; and, to determine the quality of coals and the location of mineral zones, establishing at the same time the mineral percentage content in these zones. The design of combined core-sampling sets, that of induction, lateral, acoustic, and selective gamma-gamma core sampling equipment; of instruments operating at high temperatures and pressures; of drilling core lifters of various diameters; etc., should be accelerated. It is also necessary to speed up application of advance operational methods and introduction of automation and complex mechanization, in basic and auxiliary operations.

The conference adopted a resolution, containing the following recommendations:

1) To introduce geophysical exploration methods in all types of combined geological-exploratory operations and in all of their stages, in order to achieve a higher degree of geological and economic efficiency. To make more extensive use of aerogeophysical-survey methods, and to put into effect complex mechanization and automation of labor-consuming geophysical operations, e. g. seismic exploration and core-sampling of drillings.

2) To intensify work concerned with the general application of results obtained in geological-geophysical surveys, with the study of physical properties of rocks and ores; to draw up methodical handbooks and instructions describing each geophysical method. To curtail sharply the development schedule for experimental models of new equipment; and, to reinforce design offices engaged in geophysical-equipment construction.

The conference noted the need to improve the quality of manufactured equipment, of spare parts, drilling machinery, field equipment, and special materials. Larger numbers of electronic and radio engineering specialists should be recruited by geophysical enterprises. Educational institutions should increase the number of graduating engineers and technicians in accordance with the plan calling for expansion of geophysical operations during the next seven-year period. The conference performed a large amount of useful work by outlining in its directives the basic development trends of all geophysical methods; and, by exposing the existence of serious shortcomings in the organization, conduct, and technical equipment of geophysical operations.

Participants at the conference made a number of critical observations, undoubtedly, these will be taken into consideration in order to ensure the necessary development rate of geophysical-survey methods in the very near future.